Belief-Formation in Games of Initial Play: an Experimental Investigation

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Abstract

Prior evidence suggests that individuals possess non-equilibrium beliefs in games of initial play. We investigate the belief-formation process in such settings with a lab experiment. Two main findings emerge from a novel elicitation task incentivizing participants to forecast the play of others. First, a subject anticipating a certain action generally predicts *each and every* less sophisticated action as well. Second, in the dynamic process of belief reporting, subjects order their predictions from less to more sophisticated strategies. The increasing use of non-equilibrium belief-based theories in strategic environments suggests that our results have implications in a variety of applications.

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1 Introduction

In games of initial play, equilibrium behavior is often scarce (Camerer, 2003), suggesting that when a player deliberates in such strategic settings, she errs somewhere during the process of forming beliefs and best responding to them (assuming she even thinks in such a strategic way). While others have studied this best-response assumption,[1] this paper focuses on the cognitive process a strategic player implements *before* taking an action, i.e., strategic belief-formation.

A number of questions naturally arise regarding such a process. What predictions does an individual make regarding the play of a set of opponents who have varying degrees of strategic sophistication? Does she acknowledge such strategic diversity at least up to her degree of strategic sophistication? Specifically, if she predicts a particular strategy, does she also predict the set of strategies that are less sophisticated? As a player deliberates, are types with less strategic sophistication predicted *earlier* than those with more? The answers to these questions can help shape belief-based theories of non-equilibrium strategic thinking and thus have broad implications given the increasing application of such models, theoretically and empirically.[2]

[1]See, for example, Costa-Gomes and Weizsäcker (2008) and Rey-Biel (2009).

[2]Many papers use the Level K model (Stahl and Wilson (1994), Nagel (1995)) and its point belief assumption for theoretical applications in diverse economic settings – including macroeconomic models: Farhi and Werning (2018). Empirical applications also use Level k but there are some exceptions, using the Cognitive Hierarchy model (Camerer, Ho and Chong (2004)). Robert Östling, Joseph Tao-yi Wang, Eileen Y. Chou and Colin F. Camerer (2011) partially rationalize the selection of numbers in the Swedish LUPI lottery game. Brown, Camerer and Lovallo (2013) describe moviegoer behavior more accurately than equilibrium. Hortaçu et al. (2017) estimate firms' levels of strategic sophistication and find that larger firms

There are already studies exploring *changes* in beliefs. For example, Agranov et al. (2012) find that individuals alter their beliefs and behavior depending on whether they are playing against undergraduate or graduate students. Furthermore, Alaoui and Penta (2016) find that higher incentives will induce actions associated to higher order beliefs. In contrast, we look at how beliefs form in the first place. Furthermore, with an experiment designed to show that bounded rationality is not necessarily determined by ability, Friedenberg et al. (2018) rationalize their data *ex post* by invoking non-degenerate beliefs. One of the contributions of the current paper is a design that can provide independent evidence for such diverse beliefs.

To study belief-formation, we conduct a between-subjects experiment. In our main *Beliefs* treatment, subjects predict the behavior of participants in an *Actions* treatment who have previously played a series of two-person Number Selection (NS) games. In NS games, players simultaneously select integers between 1 and some upper bound, say, 14, inclusive. If a player selects an integer, i, she earns i points. Furthermore, a player earns 100 bonus points if her number is exactly 3 less than her opponent's number and earns 35 points if her number equals her opponent's number.[3] Points for each NS game are converted to money using binary lotteries (see Roth and Malouf (1979)) to address concerns about risk preferences. In short, a subject is either paid a high reward or a low reward and the likelihood of receiving the high reward is increasing in the number of points attained.

Our NS games naturally give rise to a set of strategically related types described by the Level K model (Stahl and Wilson (1994), Nagel (1995)). The model is rooted by a naive Level 0 (L0) type and the Level $k \geq 1$ strategy is obtained by taking the best-response to

engage in more sophisticated reasoning compared to smaller firms.

[3]In some games, the "undercutting" distance is 4, not 3. Our NS games are most similar to the Generalized Centipede (GC) games from Fragiadakis, Knoepfle and Niederle (2017). These GC games in turn are inspired by the two-person 11-20 Money Request game from Arad and Rubinstein (2012) that has spurred additional papers such as Goeree, Louis and Zhang (2017) and Alaoui and Penta (2016).

the Level $k-1$ action. In NS games, an L0 player is expected to select the upper bound of the guessing range since this yields the highest number points when ignoring opponent play.[4] In the previously discussed NS game, the L0 action is therefore to select the upper bound of 14. An L1 player, then, who assumes her opponent is a Level 0 type, is incentivized to choose 11 (to earn 11 points plus the 100-point bonus). For $0 \leq k \leq 3$, the Level k strategy in this particular NS game is given by $14-3k$. For $k' \geq 4$, the Level k' strategies all equal 2, which is also an equilibrium action.[5] In addition, the game parameters are chosen so that (i) there is separation between L0 through L4 strategies, (ii) the L3 strategy is never an equilibrium action and (iii) the best response to *any* convex combination of Level-k beliefs is a Level-k action. Under (i), (ii) and (iii), we obtain a clear lens through which to study formation in our *Beliefs* treatment.

In our main treatment, each *Beliefs* participant is tasked with predicting which actions were played by a set of 20 *Actions* subjects in each of the NS games. To state the corresponding actions for a particular NS game, a *Beliefs* participant constructs (on the computer) a 20-box histogram over a horizontal axis depicting the game's guessing range (i.e. the game's set of pure strategies).[6] For constructing a given histogram, h, a subject earns p points, where p is the number of boxes that overlap with "dots" when h is superimposed with the 20-dot "true" histogram, i.e., the one describing the choices made by the 20 *Actions* subjects. A binary lottery approach similar to that of the *Actions* treatment is then used to translate points into money: earning p points from a histogram yields a *Beliefs* participant a high reward with probability $p/20$ and a low reward otherwise.

In our Actions treatment 70% of choices are consistent with some Level $k \leq 3$ actions.

[4] An alternative Level 0 specification is uniform-random play. Under the risk-neutral preferences induced via our binary lotteries approach, both of these L0 specifications give rise to a best-response of guessing 11 in this example, thus, both rules lead L1 players to the same number.

[5] In this game, (1,1), (2,2) and (3,3) are the three pure strategy Nash equilibria.

[6] Carpenter, Graham and Wolf (2013) also use histograms in a belief elicitation task but a subject's earnings depend on her histogram as well as those of others. In this paper, the latter has no influence.

With what success does a *Beliefs* participant in predict these four opponent types? Using the set of 11 histograms that a *Beliefs* subject constructs, we use the Akaike Information Criterion (AIC) to find the best strategic thinking model fitting a subject's data, controlling for additional parameters. Specifically, we take all possible 16 subsets of the the four-membered set of Level $k \leq 3$ strategies and assign to each subject the model with the best fit.

We obtain several results from the AIC analysis. First, 5 of our 81 *Beliefs* participants fail to significantly predict any Level k type for $k \in \{0, 1, 2, 3\}$.[7] Of the 76 remaining participants, 15 (20%) anticipate unsystematic subsets of lower types while 61 (80%) anticipate every lower type: {Level 0 , ... , Level $k-1$} for some k in $\{1, 2, 3, 4\}$. This result is not driven by low levels of k. In fact, 7, 9, 21 and 24 subjects predict the set of Level 0 through $k-1$ types for $k = 1, 2, 3, 4$, respectively. This substantial frequency of subjects classified as high levels according to AIC seems at odds with previous results showing that most players appear to be low levels. However, strategies' relative rates of predictability are individually in line with prior work: the Level k actions for $k = 0, 1, 2, 3$ are anticipated by 85, 76, 65 and 42 percent of *Beliefs* types, respectively.

In addition to obtaining the distribution of 20 boxes in a *Beliefs* subject's histogram for a given NS game, we record the *order* in which the 20 final boxes are arranged, unbeknownst to subjects.[8] We view this "belief-tracking" procedure as relating to studies using "eye-tracking" to record where subjects direct their attention (see, e.g., Wang, Spezio and Camerer (2010)) as well as studies where subjects must actively "open" boxes to observe payoffs from certain strategy profiles (see, e.g., Costa-Gomes, Crawford and Broseta (2001)). The main finding from this order analysis is in concordance with the stepwise na-

[7]More generally, one concern is misspecification. In our Results section, we provide support for the claim that there is very little room for misspecification in our data.

[8]Because payoffs depend only on final histograms, our instructions and interface make no mention of recording the order used to arrange boxes, allowing for a minimally invasive order tracking procedure.

ture of Level K reasoning: when a box is placed on a Level k strategy, the subsequent box is significantly more likely to be placed on that same strategy or the Level $k+1$ strategy compared to using a random ordering of boxes to arrive at the same final histogram.

As for the remainder of the paper, Section 2 presents the experiment and underlying theory, Section 3 discusses the results. While *Beliefs* participants ultimately predict different numbers of opponent types, the processes of belief-formation *up to* the number of predicted types are similar: Level 0 actions are predicted first, followed by L1, etc, until predictions stop at some Level $k \leq 3$. Section 4 concludes.

2 The Experiment

The experimental design includes two treatments. In the *Actions* treatment, subjects are asked to a play a series of Number Selection (NS) games designed to focus behavior onto the Level $k \leq 3$ actions in the Level K model. In the *Beliefs* treatment, separate subjects (drawn from the same population) are incentivized to predict the choices made in the *Actions* treatment.[9] Obtaining such beliefs allows us to identify the sets of Level k strategies predicted. In addition, the recording of the order in which beliefs are expressed allows us to investigate whether lower Level k strategies are, as one might expect, predicted earlier than higher ones.

[9] Alternatively, we could have asked subjects to predict the results of prior experiments. We believe, however, that our specific beliefs questions are most easily addressed using NS games and that having subjects from both treatments come from the same pool simplifies one aspect of the generally complex problem of belief-elicitation. Lastly, while we could have implemented a within-subject a design, we choose our between-subjects approach to mitigate concerns of spillover effects, fatigue and hedging (see, for example Costa-Gomes and Weizsäcker (2008)).

2.1 *Actions* treatment: Number Selection (NS) Games

In a generic NS game, g, a player i and her opponent simultaneously select integers, n_i and n_{-i}, respectively from a common range $R_g = \{1, 2, \ldots, UB_g\}$, where UB_g is the game's upper bound. Player i earns n_i points automatically for selecting n_i. If n_i is exactly D_g less than n_{-i}, where D_g is g's commonly known undercutting distance, then i earns $B_g > UB_g \times D_g$ *additional* points.[10] If $n_i = n_{-i}$, then player i earns $b_g \in (UB_g - 1, B_g - D_g)$ *additional* points.[11] This payoff function is shown in Equation 1.

$$\pi_i^g(n_i, n_{-i}) = n_i + \begin{cases} B_g \text{ if } n_i = n_{-i} - D_g \\ \\ b_g \text{ if } n_i = n_{-i} \end{cases} \tag{1}$$

Strategic thinking requires a specification of non-strategic behavior. Following Arad and Rubinstein (2012), we assign UB_g as the L0 action since it is the only one that maximizes a player's number of points if he does not form beliefs about his opponent. It is straightforward to see that the best response to L0 (i.e. L1) is to undercut L0 by selecting $UB_g - D_g$. Higher types are based on this result. If a player cannot undercut his opponent, her best response is to match the other player, which leads to following Level k strategies and equilibria:

Observation 1 (Level-k Strategies). *In a Number Selection Game, g, the Level k strategy is* $\max\{UB_g - k \times D_g, \text{mod}(UB_g, D_g)\}$*, where* $\text{mod}(x, y)$ *is the remainder from* $x \div y$*.*

Observation 2 (Pure Strategy Nash Equilibria). *The set of pure strategy Nash equilibria in a Number Selection game with undercutting distance D_g is* $\{(1, 1), (2, 2), \ldots, (D_g, D_g)\}$.

[10] The restriction that $B_g > UB_g \times D_g$ is needed for Observation 3.
[11] The restrictions that $b_g < B_g - D_g$ and $b_g > UB_g - 1$ is needed for Observation 2.

While we take UB_g to be the L0 strategy in NS games, other L0 specifications may be reasonable as well. For instance, L0 players are often assumed to select a pure strategy uniformly at random. While plausible in many other games, such a L0 specification in NS games makes the strong assumption of an indifference between choosing larger numbers that yield more points compared to smaller ones, *ceteris paribus*. Nevertheless, we design our games so that the Level $k > 0$ specifications are unaffected by these two specifications (Observation 3), reducing concerns about the explanatory power of strategic thinking theories being driven by idiosyncratic L0 specifications (see Hargreaves-Heap, Rojo Arjona and Sugden (2014)).

Observation 3 (**Robustness to L0 specification**). *For each Number Selection game, g, $UB_g - D_g$ is the unique best-response to uniform random opposition.*

Proof 1. *Against a uniform random opponent, player i's points in game g are:*

$$\Pi_i^g(n_i) \equiv \frac{1}{UB_g} \sum_{j=1}^{UB_g} \pi_i^g(n_i, j) = n_i + \frac{b_g}{UB_g} + \begin{cases} B_g/UB_g \text{ if } n_i \leq UB_g - D_g \\ 0 \text{ if } n_i > UB_g - D_g \end{cases} \tag{2}$$

From Equation 2, we see that UB_g and $UB_g - D_g$ are the only local maxima. Given that $B_g > UB_g \times D_g$, we have $B_g/UB_g > D_g$, which implies that

$$(UB_g - D_g + b_g/UB_g) + B_g/UB_g > (UB_g - D_g + b_g/UB_g) + D_g.$$

Therefore, $\Pi_i^g(UB_g - D_g) > \Pi_i^g(UB_g)$, which completes the proof.

Our NS games are robust to beliefs in yet another sense. In particular, if a player anticipates a diversity of Level k opponents, her best-response is nevertheless a Level k strategy. This is stated formally in Observation 4.

Observation 4 (Robustness to Diverse Beliefs). *Let* α_0^R *be the fraction of uniform random opponents a player* i *believes she is facing in a Number Selection Game,* g*. Let* α_k *be the fraction of Level* k *opponents that* i *believes she is facing, where the Level* k *action is given by Observation 1. If* $\alpha_0^R + \sum_{h=0}^{\infty} \alpha_h = 1$*, and if* n_i *is a best response to her beliefs, then* n_i *is one of the actions given by Observation 1.*

Proof 2. *Suppose* n_i' *is a strategy that is* not *spanned by Observation 1. Let* $\pi_i^g(n, \alpha)$ *denote* i*'s expected payoffs to selecting* n *given beliefs* α*.*

- *If* $n_i' < UB_g - D_g$*, then* $\pi_i^g(UB_g - D_g, \alpha) > \pi_i^g(n_i', \alpha)$*:*

$$\pi_i^g(n_i', \alpha) = n_i' + 135 \times \alpha_0^R / UB_g < UB_g - D_g + 135 \times \alpha_0^R / UB_g \leq \pi_i^g(UB_g - D_g, \alpha)$$

- *If* $n_i' > UB_g - D_g$*, then* $\pi_i^g(UB_g, \alpha) > \pi_i^g(n_i', \alpha)$*:*

$$\pi_i^g(n_i', \alpha) = n_i' + 35 \times \alpha_0^R / UB_g < UB_g + 35 \times \alpha_0^R / UB_g \leq \pi_i^g(UB_g, \alpha)$$

To illustrate Observation 4 with an example, consider a player in an NS game who believes in L0 and L1 opposition. If her beliefs place sufficient mass on L0, her best response is UB D (the Level 1 action). Alternatively, if she expects a large enough amount of L1 opposition, UB - 2D (the Level 2 strategy) is her best response. In sum, Observations 3 and 4 suggest that a rich set of strategic beliefs give rise to only a handful of best-responses – namely, the set of Level-k actions – in NS games.

2.2 *Beliefs* treatment: Box Arrangement (BA) Tasks

We create Box Arrangement (BA) tasks to elicit *Beliefs* subjects' predictions of *Actions* participants' choices.[12] In a BA task t_g that corresponds to NS game g with S pure strategies, each subject needs to distribute N indivisible boxes across a horizontal axis made up of the pure strategies in g. Each box represents an action chosen by a different *Actions* subject.

To describe how points are attained in a BA task, let $\mathbf{h} = \{h_1, \ldots, h_s, \ldots, h_S\}$ be the vector that defines a complete histogram, where $h_s \in \{0, \ldots, n\}$ is the number of boxes placed on pure strategy s. In addition, let $\mathbf{l} = \{l_1, \ldots, l_s, \ldots, l_S\}$ be the vector defining a histogram of "dots", where $l_s \in \{0, \ldots, n\}$ denotes the *Actions* subjects that chose strategy s (from a set of n *Actions* subjects). Thus, both the reported beliefs and the actual behavior are expressed using the same number of boxes and dots, respectively: $\sum_{s=1}^{S} h_s = \sum_{s=1}^{S} l_s = n$. The points earned in t_g are equal to the number of boxes that overlap with dots when $\mathbf{h}$ and $\mathbf{l}$ are superimposed. Equation 3 states this straightforward payoff function mathematically.

$$\pi(\mathbf{h}, \mathbf{l}) = \sum_{j=1}^{|S|} \min\{h_j, l_j\} \tag{3}$$

Since an individual does not know ex ante the realization of behavior, they need to maximize their utility given some belief system. Because beliefs are restricted to be some multiple of $\frac{1}{n}$, we call $\frac{1}{n}$ the *coarseness of the language* in our belief elicitation mechanism. Let p_i denote a subject's belief that a randomly sampled *Actions* subject will play strategy s_i in g. Thus, an individual's full beliefs are given by $p = (p_1, \ldots, p_S)$, where $\sum_{i=1}^{S} p_i = 1$ and $p_i \in [0, 1]$ for all i. Suppose that subject can make any box arrangement of n boxes and,

[12] After doing all BA tasks, subjects also do incentivized risk and relative performance tasks for further controls. Details and results for these secondary tasks appear in Appendix C.

given her beliefs p, is incentivized to report histogram h^r, where h_i^r is the number of boxes placed on strategy i. We can then define the following notion of incentive compatibility up to language coarseness $1/n$ and prove that our BA task satisfies this notion.

Definition 1. *BA tasks are incentive compatible up to language coarseness* $1/n$ *if both of the following are true for an optimal histogram* h^r *reflecting beliefs* p*:*

- $h_i^r > \lceil np_i \rceil$ *for some* i *implies* $h_j^r \geq \lfloor np_j \rfloor$ *for all* j
- $h_i^r < \lfloor np_i \rfloor$ *for some* i *implies* $h_j^r \leq \lceil np_j \rceil$ *for all* j

where $\lceil x \rceil$ *and* $\lfloor x \rfloor$ *are the ceiling and floor functions, respectively.*[13]

Theorem 1. *Each BA task is* incentive compatible up to the coarseness of the language *according to Definition 1. (See Appendix B for proof.)*

In the literature, we find two main deterministic incentive compatible methods for eliciting beliefs: proper scoring rules (Savage (1971)) – in particular, the popular Quadratic Scoring Rule (see Selten (1998) for an overview) – and the direct revelation mechanism (Karni (2009)). Although widely used, there are important practical shortcomings for both methods, namely that stakes, incentives and hedging opportunities can substantially distort reported probabilities (see Armantier and Treich (2013) for a recent review). Our elicitation mechanism differs from these standard rules in two ways.

First, payoffs are determined using binary lotteries in order to reduce the likelihood of belief formation being affected by risk preferences. Hossain and Okui (2013) and Harrison, Martínez-Correa and Swarthout (2014) find that such rules can indeed induce actions closer to risk neutrality compared to the Quadratic Scoring Rule.[14]

[13] If x is an integer, $\lceil x \rceil = \lfloor x \rfloor = x$. If x is not an integer, $\lceil x \rceil = y$ such that y is the unique integer in the set $(x, x+1)$ and $\lfloor x \rfloor = z$ such that z is the unique integer in the set $(x-1, x)$.

[14] Schlag and van der Weele (2013) discuss some of the advantages of probabilistic elicitation over deterministic elicitation.

Second, we choose a discrete reporting procedure so that i) we can cleanly analyze the order in which beliefs are stated and ii) elicit beliefs via frequencies (how many subjects out of 20 chose this number?) as oppose to probabilities (how likely is it that a given subject chose this number?). Psychologists Gigerenzer and Hoffragre (1995) have argued that such a *frequency format* corresponds to the sequential way in which information is acquired in natural sampling and, consequently, may be more intuitive than stating likelihoods in a *probability format.*[15] Frequency formats may also lead to less extreme elicited beliefs compared to probability formats. In particular, subjects asked for probabilities of a binary event may anchor their responses at 0 (did not happen) and 1 (did happen). In fact, Huck and Weizsäcker (2002) find that when subjects are asked to forecast rates of others choosing between two options, predictions follow moderate distributions.

2.3 Experimental Procedures

Experimental subjects are undergraduates and graduate students from Texas A&M University (TAMU), recruited using ORSEE (Greiner (2015)). Participants interact via a network of computers linked by z-Tree (Fischbacher (2007)) at the Economics Research Laboratory in TAMU's Department of Economics. Two 20-subject sessions, each lasting approximately 1 hour, make up the *Actions* treatment. The 81 participants in the *Beliefs* treatment are spread across 5 sessions of 14, 13, 18, 18 and 18 subjects, each taking roughly 2 hours. Average earnings are \$28.39 and \$54.07 for *Actions* and *Beliefs* subjects, respectively, including a \$5.00 show-up payment. *Actions* participants play the 11 Number Selection (NS) games shown in Table 1, where each game has a lower bound of 1, $B_g = 100$ and $b_g = 35$.

[15] In a series of experiments, they find that when information is presented in frequency format, participants submit almost double the answers consistent with Bayesian updating compared to when it is presented in a probability format. There is plenty of behavioral evidence (cited in their paper) reporting that applying Bayes rule is extremely difficult for subjects when using the probability format.

Subjects do not receive feedback between games.[16]

We use a set of design criteria to select the specific parameters for the NS games in Table 1. First, each game has at most 32 pure strategies in order to facilitate a legible display when *Beliefs* subjects construct their histograms. Second, we impose that $D_g \in \{3, 4\}$ so that Level K reasoning can only explain a fraction of strategies. As a result, we thirdly impose that upper bounds are at least 14 to ensure that the Level 0 through 3 predictions are all distinct and that none coincide with equilibria.[17]

TABLE 1.—The 11 Number Selection Games used in the experiment

Game Number (g)	1	2	3	4	5	6	7	8	9	10	11
UB_g	14	17	20	23	26	29	32	18	19	22	23
D_g				3					4		

Actions participants in a session are paired randomly and anonymously at the beginning of their session; matchings are fixed for all 11 games. NS games are presented to participants as in Figure 1. Each *Actions* subject views an NS game from the same perspective: a subject is addressed as "You" and her opponent is referred to as "The Other Participant". *Beliefs* participants are shown the same 11 NS games from Table 1, presented as in Figure 1, except that the two players are referred to as "Jack" and "Jill".

For a game, g, a *Beliefs* subject performs a BA task, t_g. When performing a BA task, a subject's screen initially shows a large, empty, rectangular area that has the game's

[16]Weber (2003) shows that subjects can learn without feedback as they gain experience with games. We investigated whether the frequencies of the actions associated to different Level $k \in \{0, 1, 2, 3\}$ change across time and find no significant differences.

[17]We believe such type separation is sufficient given that others have shown the frequencies of higher levels drop off rather quickly. For example, Crawford and Costa-Gomes (2006) and Fragiadakis, Knoepfle and Niederle (2016) classify substantially more Level $k \in \{1, 2\}$ subjects compared to Level $k \geq 3$. In addition, we also required $\{UB_g - k \times D_g\} \Cap \{1, D_g\} = \emptyset$ for all k in order to distinguish the equilibria to which Level K strategies converge from two additional equilibria: the lower bound equilibrium of (1,1) as well as the efficient equilibrium of (D_g, D_g).

The RANGE is **1 to 14** and the UNDERCUTTING DISTANCE is **3**.

You and The Other Participant are to select Numbers from the Range.

You will receive the Number you select **IN POINTS** and The Other Participant will receive the Number they select **IN POINTS**.

You will receive **100 BONUS POINTS** if your Number is **exactly 3 less** than The Other Participant's Number.

The Other Participant will receive **100 BONUS POINTS** if their Number is **exactly 3 less** than your Number.

If You and The Other Participant select the **same Numbers**, you will each earn **35 BONUS POINTS**.

1	2	3	4	5	6	7	8	9	10	11	12	13	14
☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐

FIGURE 1.—How subjects view Number Selection Games

range of numbers, $R_g = \{1, 2, \ldots, UB_g\}$, strung along its lower horizontal edge. A green, upwards-pointing, arrow button rests underneath each number in R_g. Clicking the green arrow button under a number $n \in R_g$ adds a blue box above n. When one or more blue boxes are placed above n, a red, downwards-pointing, arrow button shows below n's green arrow button. Clicking n's corresponding red arrow button removes the top-most blue box that are stacked above n. A subject can click green and red arrow buttons without any other restrictions, allowing her to freely build (and revise) her histogram until she is ready to submit it. Between the game's range and the green arrow buttons, a counter shows the number of boxes resting above each number in R_g.

Subjects click the arrow buttons to allocate their 20 blue boxes across the strategies in

R_g to express how they believe the 20 participants from a previously run *Actions* session made their choices in g. For instance, if a *Beliefs* participant believes that two *Actions* subjects chose $1 \in \{R_g\}$, she would place two blue boxes above the number "1" – as shown in Figure 2. We provide subjects with a plastic transparency depicting the example Box Arrangement from Figure 2 and a sheet of standard white paper showing the example Dot Arrangement from Figure 3 (yellow dots) of hypothetical play by 20 *Action* subjects. We explain that if subjects superimpose the two arrangements as in Figure 3, they earn p points for having p boxes that overlap with dots. Keeping with our previous example using the number 1, the Dot Arrangement indicates three subjects choosing 1 while the Box Arrangement shows that only two of these dots are predicted with boxes. In total, Figure 3 shows $p = 14$ boxes overlapping with dots.

While the points earned do not depend on the order in which boxes are arranged, one novel aspect of our experimental design is that the order in which *Beliefs* participants arrange the boxes that make up their final histograms is recorded. The interface and instructions do not make order salient in any way, making the tracking minimally invasive. We thus interpret the sequence of choices as revealing information about the order in which beliefs come to subjects' minds.

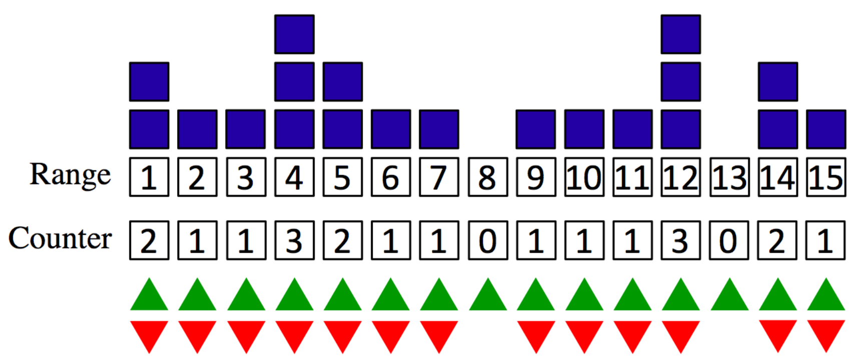

FIGURE 2.—a histogram for a fictitious Box Arrangement Task

In both treatments, instructions are read out loud and subject comprehension is rein-

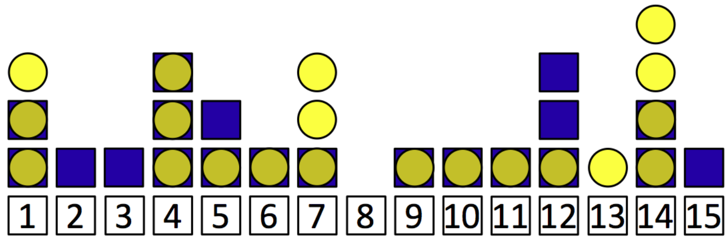

FIGURE 3.—a fictitious Dot Arrangement overlaid with the histogram from Figure 2

forced using on-screen understandings tests (see Appendix E). A subject cannot proceed past a question until it is answered correctly. To start the actual experiment, all subjects need to complete the corresponding understanding test. Subjects receive no feedback whatsoever until the end of the entire study.

As mentioned above, the point payoffs in each NS game and BA task are converted to money (at the end of the experiment) using separate and independently run binary lotteries (Roth and Malouf (1979)). If a subject earns p points in an NS game, the corresponding lottery pays \$5 with probability $p/150$ and \$1 with probability $1-p/150$. If a subject earns p points in a BA task, she earns \$5 with probability $p/20$ and \$1 otherwise.

3 Results

The results of our NS games show a concentration of both behavior *and* beliefs on the four Level $k \leq 3$ strategies. Table 2 shows that Level $k \geq 4$ and equilibrium choices are far less common. In Appendix D, we plot the frequencies of all choices in all games for both treatments, which clearly show how the data "spike" on Level $k \leq 3$ choices. To get a better perspective of the percentages in Table 2, the rightmost column lists the percentages of expected play under hypothetical uniform-random decision-making.

To investigate our questions relating to belief-formation, we consider *Beliefs* subjects' final histograms as well as the sequences of belief reporting. The following subsections

	Actions treatment	*Beliefs* treatment	Random
Level $k \in \{0, 1, 2, 3\}$	70	42	18
Level $k \in \{4, 5, 6, 7, 8, 9\}$	1	5	8
Equilibrium	3	9	15

TABLE 2.—Percentages of various observed and random hypothetical choices.

present the results of these analyses.

3.1 Classifying *Beliefs* Participants using the Akaike Information Criterion (AIC)

Each *Beliefs* participant constructs 11 histograms of 20-boxes each, stating her predictions of the play by 20 *Actions* subjects in the 11 Number Selection (NS) games. On average, *Beliefs* participants earn 9 points per histogram with a standard deviation of 2 points. In each NS game, the Level 0 through 3 strategies are distinct and equilibria coincide with the Level k strategies no earlier than at Level 4. This allows for clean identification of beliefs over Level k actions for $k \in \{0, 1, 2, 3\}$. We proceed by making use of the Akaike Information Criterion (AIC) to determine which subset of types from the set T = {L0, L1, L2, L3} provides the best fit for each *Beliefs* participant, partitioning these 81 subjects across the 16 models that arise from taking all possible subsets of T.

Specifically, we estimate 16 linear regressions (one per model) via maximum likelihood for each of the 81 *Beliefs* subjects. Each model also includes a constant and set of game dummies controlling for fixed effects. For example, the most inclusive model, M_{16}, which represents a *Beliefs* participant who predicts Level 0 through Level 3 for the *Actions* subjects, is estimated as

$$M_{16} : \mathit{boxes} = \beta_0 + \beta_1 L0 + \beta_2 L1 + \beta_3 L2 + \beta_4 L3 + \sum_{i=1}^{10} \beta_i (\mathit{game}_i) + \epsilon,$$

where *boxes* is the number of boxes on a column in a subject's histogram, $game_i$ is a set of dummy variables for each game (to control for idiosyncratic game-specific characteristics), Lk for $k \in \{0, 1, 2, 3\}$ are indicators for the Level k choices and ϵ is an error term.[18] The full set of regression models M_1 through M_{16} are listed in Appendix A. The regression results for each subject and model yield an AIC statistic

$$AIC = n\ln(RSS/n) + 2K,$$

where n is the sample size,[19] K is the number of estimable parameters (degrees of freedom) and RSS is the residual sum of squares. Following Hurvich and Tsai (1989), we account for small sample sizes by using a corrected AIC statistic

$$AICc = AIC + \frac{2K(K+1)}{n-K-1},$$

where AIC, n and K are defined as above. Of a *Beliefs* subject's 16 $AICc$ scores (one per model), the smallest such score indicates the model that best fits her choices, after controlling for the number of parameters to be estimated. For example, the best fit for an individual may be the model that includes $L0$ and $L2$ as the only Level k regressors. In this case, the subject would be classified as the AIC type that predicts only $L0$ and $L2$ opponent types. Table 3 shows the subset of Level $k \leq 3$ opponent types predicted by each AIC type as well as the number of *Beliefs* subjects classified as each AIC type. For example, an AIC9 type predicts L1 and L2 and hence, these are the only strategies denoted using a • symbol in the table. Looking at the entry in the bottom-row of the AIC9 column,

[18]As long as one is willing to assume that the error term is normally distributed, the results of the maximum likelihood method presented in the main text should tend to be equivalent to those using the least square method to test the different hypotheses. In particular, the inferences of the classification below do not change for 73% of our subjects when we use OLS instead. These robustly classified subjects also present a higher adjusted R^2 on average than the remaining subjects.

[19]In our case, $n = 243$ because this is the sum of the strategy sets across the 11 NS games.

the 1 indicates that only one subject is classified as an AIC9 type.

Table 3.—Subject Classification using the Akaike Information Criterion

	AICm type for $m \in \{1, \dots, 16\}$																
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	total
L0		•				•	•	•				•	•	•		•	69
L1			•			•			•	•		•	•		•	•	62
L2				•			•		•		•	•		•	•	•	53
L3					•			•		•	•		•	•	•	•	34
total	5	**7**	1	1	1	**9**	2	2	1	0	0	**21**	3	1	3	**24**	

For a given AIC type, each • denotes a prediction of the corresponding Level K strategy. For example, AIC2 types predict Level 0 players only. The four most common AIC types are AIC2, AIC6, AIC12 and AIC16. The total numbers of participants classified as each of these four types are bolded in the bottom row. The rightmost column denotes the total number of subjects who predict each Level k strategy.

Model 16 captures the most participants, followed by Models 12, 6 and 2. Interestingly, these models (and only these models) respect the following property: when k types are predicted, they are the Level 0 through $k-1$ types. Put differently, when stating beliefs, participants tend to predict the set of opponent types up to their cognitive limits (or up to their beliefs over the cognitive limits of others). These "principal" models are very successful in describing *Beliefs* participants:

Result 1. *While the principal models comprise only 1/4 of the set of AIC models, they capture 3/4 of* Beliefs *participants.*

Interestingly, as the four models in Result 1 acquire more sophisticated belief structures, they classify increasing numbers of participants. At first glance, this seems to be at odds with previous results identifying lower types more frequently. However, if we consider how frequently each Level k action is predicted in the AIC estimation, we see that this intuition is restored: the rightmost column of Table 3 shows that fewer and fewer subjects predict Level k opponent types as k increases.

One potential concern regarding the AIC estimation we perform is misspecification. While Table 2 suggests this is unlikely in our case, for robustness we check for the presence of actions or beliefs that are *not* captured by Level k thinking. To do so, we check if the frequency of an action predicts the probability of being a Level K action in the *Actions* treatment. Similarly, we also check if higher columns appear in Level K actions in the *Beliefs* treatment. The corresponding results using OLS and Probit appear in Table 4 (*Actions*) and Table 5 (*Beliefs*).

TABLE 4.—Commonly Taken Actions are More Likely Level K

	Indicator of a Level K Action	
	(OLS)	(Probit)
Frequency of Action in a Game	0.05^{***}	0.15^{***}
(in *Actions* stage)	(0.00)	(0.03)
Constant	0.35^{***}	-0.41^{***}
	(0.03)	(0.09)
Observations	286	286

TABLE 5.—Commonly Expressed Beliefs are More Likely Level K

	Indicator of a Level K Belief	
	(OLS)	(Probit)
Height of Column	0.02^{***}	0.04^{***}
(in $Beliefs$ stage)	(0.00)	(0.01)
Constant	0.41^{***}	-0.23^{***}
	(0.00)	(0.01)
Observations	23,166	23,166

The results show that, in all regressions, there is a significant and positive effect between the frequency of the actions or the height of a column and the action being Level K. Choices that are not explained by the Level K attract fewer boxes in the *Beliefs* treatment and fewer subjects in the *Actions* treatment. Result 2 summarizes these results about model specification.

Result 2. *The regression results from Table 4 (*Actions*) and Table 5 suggest the AIC estimation we perform leaves little room for misspecification.*

3.2 Considering the Order in Which *Beliefs* Participants State Predictions

The order of appearance of final boxes arranged in a histogram can provide insights as how beliefs about types are formed. For instance, the iterative nature of Level-k reasoning suggests that actions with a lower k will be expressed earlier. Hence, we consider 16 (4×4) pairs of transitions – between Level k and k' choices for $k, k' \in \{0, 1, 2, 3\}$. We present the corresponding Markov transition sub-matrix in Table 6. For example, the percentage of L1 choices that appear after a L0 choice in the final order is 2.92%.

Under the null hypothesis that this transition was i.i.d., the entries in the cell should be equal to the observed empirical frequency of choosing Level k' conditional on having chosen Level k earlier. If subjects make some transition more often than under i.i.d., we should expect higher percentages. In that case, a one-tailed binomial test indicates if the frequency observed is significantly higher than the frequency expected under the null hypothesis of an i.i.d. process. Results are reported in Table 6. (Notice that the percentage mass needs to be equal to 100%. So, if one entry is higher than expected, then, at least, another entry needs to be lower than expected. We indicate these entries in parenthesis in Table 6).

TABLE 6.—Transition submatrix between Level k and k'

		Choice in $t+1$			
		L0	L1	L2	L3
Choice in t	L0	7.44***	2.92***	(0.38)	(0.05)
	L1	(1.26)	8.29***	2.00***	(0.14)
	L2	(0.38)	(0.61)	3.77***	1.25***
	L3	(0.11)	(0.14)	(0.34)	1.26***

First, we note that the main diagonal cells are all positive and significant at the 1% significance level, indicating that a box placed in a category is significantly more likely to be preceded by a box in that same category. Furthermore, we also see individuals placing boxes in Level k before $k+1$. In fact, every entry showing these transitions are also positive and significant at the 1% significance level.

Result 3. *Beliefs transition significantly more frequent from a Level k action to the same level action or one higher level action.*

Table 6 looks only at transitions between Level k choices for $k \leq 3$. More generally, we can partition all choices into three mutually exclusive groups: Level k, non-Level k equilibrium (EQ) and "Remaining actions". All subsequent choices then fall into one of five categories. Each of these categories are listed in each row in Table 7 and can be described as follows. First, subjects can place a box on the same strategy as before (first row of entries in the table). Second, subjects can choose the action that is the unique best-response to previous action. (For this reason, when the t choice is an EQ, a $t+1$ choice is the same strategy if and only if it is also a best-response, merging the 26.44 value between the two rows.) Third, subjects can choose other Level k actions that are not the current action or a best response to current action. Fourth, when subjects choose a non-Level k EQ action, they can decide to switch to another non-Level-k equilibrium. Any remaining choice is included under the label "Other Remaining actions". We present the percentage of actions found in each group and category in Table 7.

There are some interesting features in Table 7. The percentage of choices that are the same as or best responses to previous choices after Level k decisions is more than double in comparison to non-Level k EQ[20] and almost triple compared to Remaining actions.[21]

[20] This comes from comparing $60.9 = 44.16 + 16.75$ to 26.44.

[21] This comes from comparing $60.91 = 44.16 + 16.75$ to $20.64 = 14.85 + 5.79$.

TABLE 7.—Classification of subsequent choices given the precedent choice

% of choices in $t+1$	Choice in t		
	Level k	non-Level-k EQ	Remaining actions
Same action	44.16	26.44	14.85
Best-response	16.75		5.79
Other Level k	10.12	25.91	31.30
Other non-Level-k EQ	3.50	12.60	4.75
Other Remaining actions	25.47	35.05	43.31
Total	100	100	100

The opposite is true when considering the percentages of choice in $t+1$ that are "Other Level-k". Non-Level-k equilibrium and "Remaining actions" in t are double (25.91%) and triple (31.30%) the percentage of those after a Level-k action (10.12%). Other non-Level-k equilibrium is marginally selected unless the previous choice was a non-Level-k equilibrium.

In short, if the choice in t is Level-k, the chance of being a Level-k action again in $t+1$ is $71.03\% = 44.16 + 16.75 + 10.12$. Even when choices in t are not Level-k, the probability of being a Level-k in $t+1$ is substantial 25.91% and 31.3% for non-Level-k equilibrium and Remaining actions, respectively.

4 Discussion

In this paper, we focus on the cognitive process a strategic player undergoes when forming beliefs over opponent behavior in games of initial play. Subjects in an *Actions* treatment play Number Selection (NS) games that induce a substantial proportion of Level $k \leq 3$ behavior, allowing us to use an AIC estimation procedure to identify which subset of these four types are predicted by each separate subject in our *Beliefs* treatment. We gain further insight into belief-formation by recording the order in which discrete beliefs are expressed. In sum, we find that *Beliefs* participants first predict Level 0 opponents, followed by L1 and so on, up to some stopping point. To our knowledge, we are the first to document this pattern of belief-formation in games of initial play.

What determines the particular level at which an individual "stops" remains uncertain. A natural explanation, of course, is cognitive limitations. Alternatively, players may believe others are cognitively limited or that others believe others are and so on. An interesting avenue for future research would be to disentangle these possibilities; our experiment was not designed to do so. Prior work does suggest, however, that individuals stop their iterative reasoning before running up against their cognitive limits, at least to some extent.[22]

With respect to existing belief-based theories of non-equilibrium behavior in games of initial play, we see our results as identifying a modeling tradeoff. On one hand, the standard Level K model generates the simplest stepwise predictions in games and has had been extremely successful in explaining behavior in a number of games. Fragiadakis, Knoepfle and Niederle (2016) show, for example, that the decision rules of strategic players almost exclusively follow standard Level K predictions. On the other, the assumption of point-beliefs may be an oversimplification in some games. In fact, behavioral theories indeed exist that model players as having non-degenerate beliefs – for example, Stahl and Wilson (1995) and Camerer, Ho and Chong (2004). Even in the p-beauty contest games that helped establish Level K, substantials fractions of choices often fail to coincide with Level K predictions; non Level K choices may, however, result from a player best-responding to a *distribution* of Level K opponents. Understanding what aspects of a strategic environment induce an agent to have diverse versus point beliefs remains an interesting open question

[22] For example, Fragiadakis, Knoepfle and Niederle (2016) show that a player who formerly played some Level k strategy consistently can best-respond to her past-self and play the Level $k+1$ strategy consistently. Agranov et al. (2012) finds that individuals alter their behavior depending on whether they are playing against undergraduate or graduate students. Georganas, Healy and Weber (2015) provide subjects with information about opponent performance on a variety of quizzes and find that this information can affect choices as well. Slonim (2005) shows that experienced players adjust their decisions depending on the experience level of their opponents.

for future work.

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Appendix

Appendix A: Regression Models used in AIC Estimation

$M_1 : boxes = \beta_0 + \sum_{i=1}^{10} \beta_i(game_i) + \epsilon$

$M_2 : boxes = \beta_0 + \beta_1 L0 + \sum_{i=1}^{10} \beta_i(game_i) + \epsilon$

$M_3 : boxes = \beta_0 + \beta_1 L1 + \sum_{i=1}^{10} \beta_i(game_i) + \epsilon$

$M_4 : boxes = \beta_0 + \beta_1 L2 + \sum_{i=1}^{10} \beta_i(game_i) + \epsilon$

$M_5 : boxes = \beta_0 + \beta_1 L3 + \sum_{i=1}^{10} \beta_i(game_i) + \epsilon$

$M_6 : boxes = \beta_0 + \beta_1 L0 + \beta_2 L1 + \sum_{i=1}^{10} \beta_i(game_i) + \epsilon$

$M_7 : boxes = \beta_0 + \beta_1 L0 + \beta_2 L2 + \sum_{i=1}^{10} \beta_i(game_i) + \epsilon$

$M_8 : boxes = \beta_0 + \beta_1 L0 + \beta_2 L3 + \sum_{i=1}^{10} \beta_i(game_i) + \epsilon$

$M_9 : boxes = \beta_0 + \beta_1 L1 + \beta_2 L2 + \sum_{i=1}^{10} \beta_i(game_i) + \epsilon$

$M_{10} : boxes = \beta_0 + \beta_1 L1 + \beta_2 L3 + \sum_{i=1}^{10} \beta_i(game_i) + \epsilon$

$M_{11} : boxes = \beta_0 + \beta_1 L2 + \beta_2 L3 + \sum_{i=1}^{10} \beta_i(game_i) + \epsilon$

$$M_{12} : boxes = \beta_0 + \beta_1 L0 + \beta_2 L1 + \beta_3 L2 + \sum_{i=1}^{10} \beta_i (game_i) + \epsilon$$

$$M_{13} : boxes = \beta_0 + \beta_1 L0 + \beta_2 L1 + \beta_3 L3 + \sum_{i=1}^{10} \beta_i (game_i) + \epsilon$$

$$M_{14} : boxes = \beta_0 + \beta_1 L0 + \beta_2 L2 + \beta_3 L3 + \sum_{i=1}^{10} \beta_i (game_i) + \epsilon$$

$$M_{15} : boxes = \beta_0 + \beta_1 L1 + \beta_2 L2 + \beta_3 L3 + \sum_{i=1}^{10} \beta_i (game_i) + \epsilon$$

$$M_{16} : boxes = \beta_0 + \beta_1 L0 + \beta_2 L1 + \beta_3 L2 + \beta_4 L3 + \sum_{i=1}^{10} \beta_i (game_i) + \epsilon$$

Appendix B: Proof of Theorem 1

Let h be a histogram a subject can build and h_i denote the number of boxes that are placed on s_i. Suppose h_1 and h_2 are such that

- $p_1 p_2 > 0$,
- $\lceil np_1 \rceil < h_1 \leq n$,
- $0 \leq h_2 < \lfloor np_2 \rfloor$.

Then, that subject would earn strictly higher expected payoffs (given p) by submitting h', which is identical to h, except that

- $h'_1 = h_1 - 1$
- $h'_2 = h_2 + 1$

Define $\widehat{p_1} = \lceil np_1 \rceil / n$. Then, $n\widehat{p_1} = \lceil np_1 \rceil \geq np_1$. Similarly, if we define $\widehat{p_2} = \lfloor np_2 \rfloor / n$, we know that $n\widehat{p_2} = \lfloor np_2 \rfloor \leq np_2$. Thus, $\widehat{p_1} \geq p_1$ and $\widehat{p_2} \leq p_2$. Then,

- $h_1 > n\widehat{p_1}$
- $h_2 < n\widehat{p_2}$

Note that h and h' differ only by a "marginal box" that will have a greater likelihood of overlapping with the behavior of an actual subject when placed on strategy 2 versus strategy 1 if and only if $P(J \geq h_2 + 1) > P(I \geq h_1)$, where $J \sim B(n, p_2)$ and $I \sim B(n, p_1)$ and $B(n, p)$ is the standard binomial distribution. To prove that this inequality holds, we will instead show that $P(\widehat{J} \geq h_2 + 1) > P(\widehat{I} \geq h_1)$, where $\widehat{J} \sim B(n, \widehat{p_2})$ and $\widehat{I} \sim B(n, \widehat{p_1})$. This will be sufficient because $\widehat{p_1} \geq p_1$ and $p_2 \geq \widehat{p_2}$ imply that $P(\hat{I} \geq h_1) \geq P(I \geq h_1)$ and $P(J \geq h_2 + 1) \geq P(\hat{J} \geq h_2 + 1)$. As we proceed, we will make use of the following:

1. If $X \sim B(n, p)$ and np is an integer, np is the unique median of X.
2. If m is the median of $X \sim B(n, p)$, then $\min\{P(X \leq m), P(X \geq m)\} \geq 1/2$.

Since $n\widehat{p_2}$ is the median of $\widehat{J}$, we know that $1/2 \leq P(\widehat{J} \geq n\widehat{p_2})$. Because $n\widehat{p_2} \geq h_2 + 1$, we know that $P(\widehat{J} \geq n\widehat{p_2}) \leq P(\widehat{J} \geq h_2 + 1)$. Thus, $\boxed{1/2 \leq P(\widehat{J} \geq h_2 + 1)}$. Now, since $h_1 \geq n\widehat{p_1} + 1$, we know $P(\widehat{I} \geq h_1) \leq P(\widehat{I} \geq n\widehat{p_1} + 1)$. Since $n\widehat{p_1}$ is the median of $\hat{I}$, we know $1/2 \leq P(\hat{I} \leq n\widehat{p_1}) < P(\hat{I} \leq n\widehat{p_1} + 1)$. Combining this with the assumption that $P(\hat{I} \geq n\widehat{p_1} + 1) \geq 1/2$ would imply $n\widehat{p_1} + 1$ is the median of $\hat{I}$. Since we know the unique median of $\hat{I}$ is $n\widehat{p_1}$, the assumption that $P(\hat{I} \geq n\widehat{p_1} + 1) \geq 1/2$ must be false. Thus, it must be that $P(\hat{I} \geq n\widehat{p_1} + 1) < 1/2$ and therefore, $\boxed{P(\hat{I} \geq h_1) < 1/2}$. Combining the boxed inequalities, we obtain $P(\hat{I} \geq h_1) < P(\hat{J} \geq h_2 + 1)$, which completes the proof.

Appendix C: Confidence and Risk

4.0.1 Relative Performance (RP) Questions

After a *Beliefs* subject i completes all 11 BA tasks, she performs 11 corresponding Relative Performance (RP) questions. For the RP question q_g corresponding to BA task t_g and NS game g, subject i is shown g *as well as* the histogram she constructed in t_g. Participants are not shown any histograms that were made by any other *Beliefs* subjects. Subject i is informed of the number of subjects in her lab session and is asked to estimate how many participants in her session she believes earned *strictly more* points than she did in t_g. For q_g, subject i earns \$5 for a correct answer and \$1 otherwise. The RP questions are intended to estimate subjects' levels of confidence in the histograms created in the BA tasks.

4.0.2 Bomb Risk (BR) Decisions

After subjects in the *Beliefs* treatment perform their RP questions, they make a Bomb Risk (BR) decision (adapted from Crosetto and Filippin (2013)) as a quick measure of their risk attitudes. The BR decision is very straightforward. There are 100 treasures chests, one of which contains a bomb. The subject chooses, m, the number of (randomly picked) chests it would like the computer to open. If the bomb is in an opened chest, the subject earns nothing (which occurs with a $m/100$ chance). Otherwise, the bomb-filled chest is *not* opened and the subject earns $m/10$ dollars.

4.0.3 Column Heights, Confidence and Risk

In this subsection, we look at the relationship between the risk preferences, confidence and the heights of the columns built by subjects in the Box Arrangement Tasks.

A subject with a belief that places a high probability on some (or any) particular action may fail to report an accurate representation of her beliefs in our Box Arrangement Task if our belief elicitation payment scheme is not successful at inducing risk neutrality. In such a case, a risk averse individual may feel incentivized to construct low columns across a large set of actions. In Table C1, specification (2), we see that it is not the case: individuals, who leave more boxes closed in the BR task (more risk averse subjects), do not construct Box Arrangements of shorter columns. This provides suggestive evidence that the binarized payment rule in our belief elicitation procedure successfully induces risk neutrality.

As we know from Result 2, the height of a column is highly correlated with being a Step k Action. Specification (1) in Table C1 shows that the heights of the columns are also correlated with the average confidence level of a subject who constructs them. In Table C1, column (3), we see that, even after controlling for the variation in risk preferences, the effect of confidence remains strong and significant.

Table C1: Column Heights, Confidence and Risk

	Sum of Squared Column Heights		
	(1)	(2)	(3)
Average Confidence	1061.26***		1046.75**
	(392.20)		(396.4166)
Boxes Left Closed		-1.59	-.91
		(2.61)	(2.53)
Constant	-51.01	735.13***	6.71
	(264.46)	(147.30)	(310.28)
Observations	81	81	81

The table shows the results of linear regressions with standard errors are clustered at the individual level and shown in parentheses. The left-hand side variable (Sum of Squared Column Heights) is self-explanatory: we square the height of each column a subject constructs in her histograms and find the total. *Average Confidence* is the average of a subject's 11 measures of normalized confidence, where a participant's normalized confidence for a histogram is equal to $(a - b)/a$ where a equals the number of *other* participants in one's session and b equals the number of other participants that the subject believes has strictly more overlapping boxes compared to her. *Boxes Left Closed* is very self-explanatory; it is the number of treasure chests that one leaves closed in the Bomb Risk Decision. $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$.

Appendix D: Actions and Beliefs in All Games

Here we present the aggregate choices from NS game and BS task. One can clearly see "spikes" in the data at the Level $k \leq 3$ strategies.

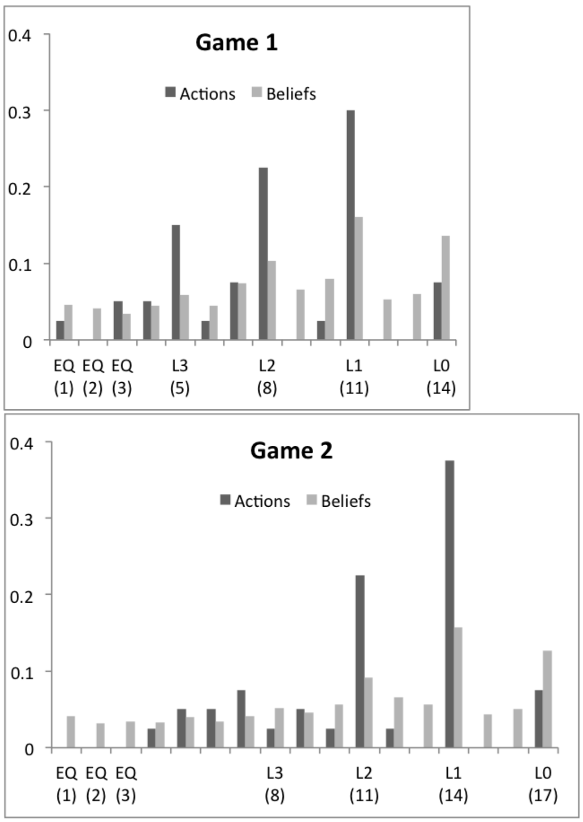


FIGURE 4.—Relative proportions of decisions in each treatment and game. (Bars in a given chart of a given treatment sum to 1.)

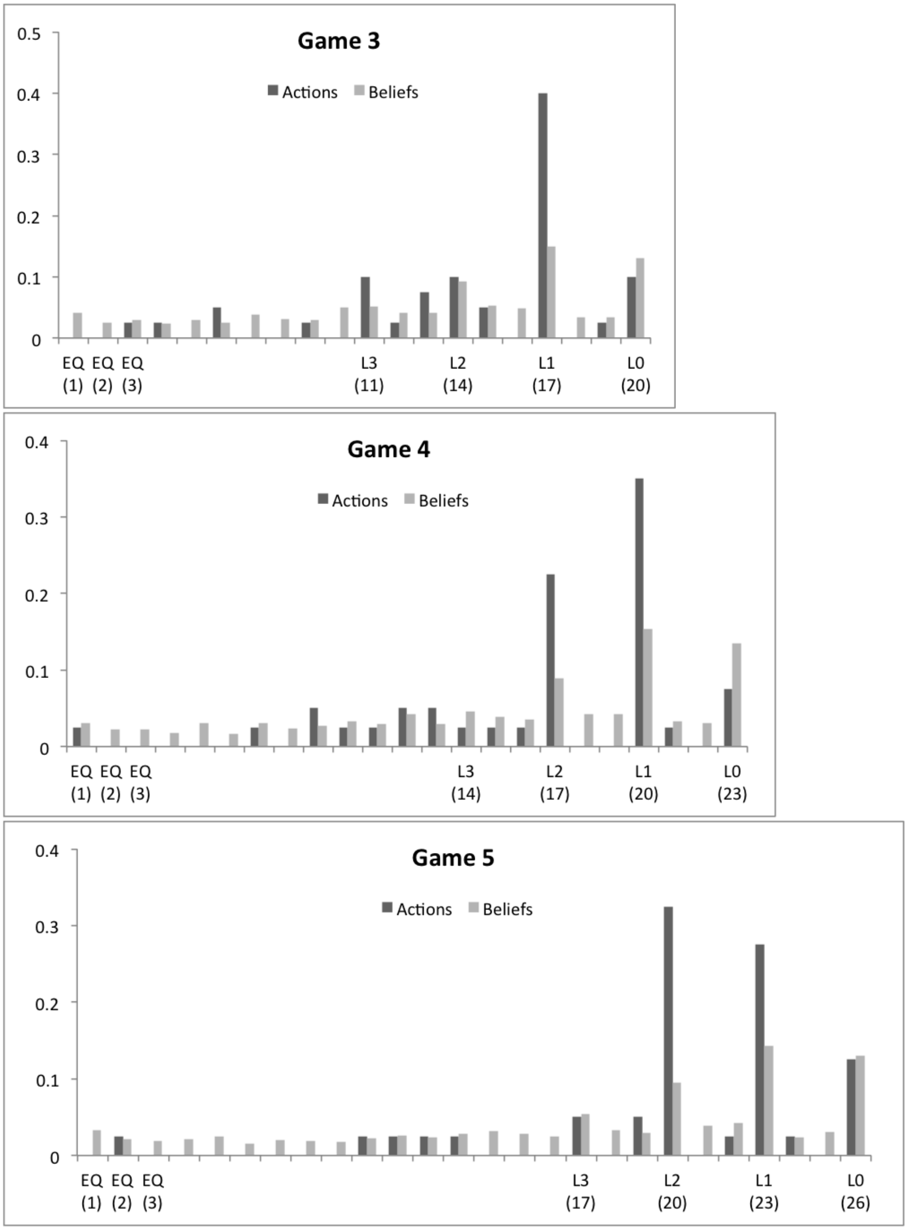
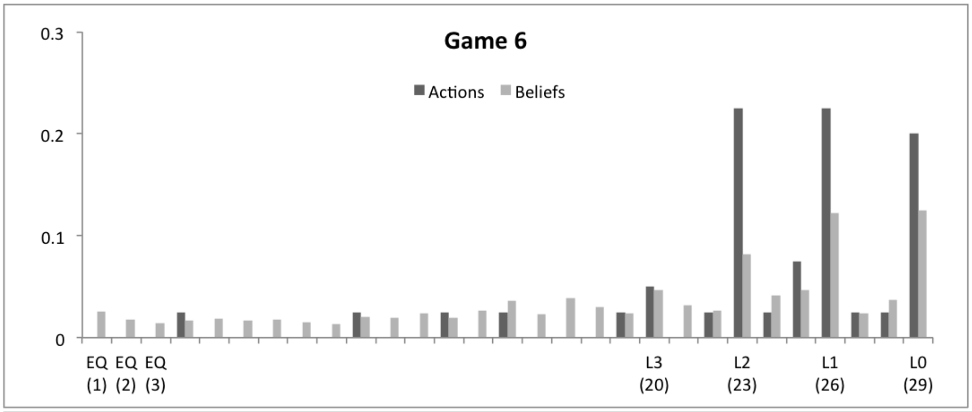
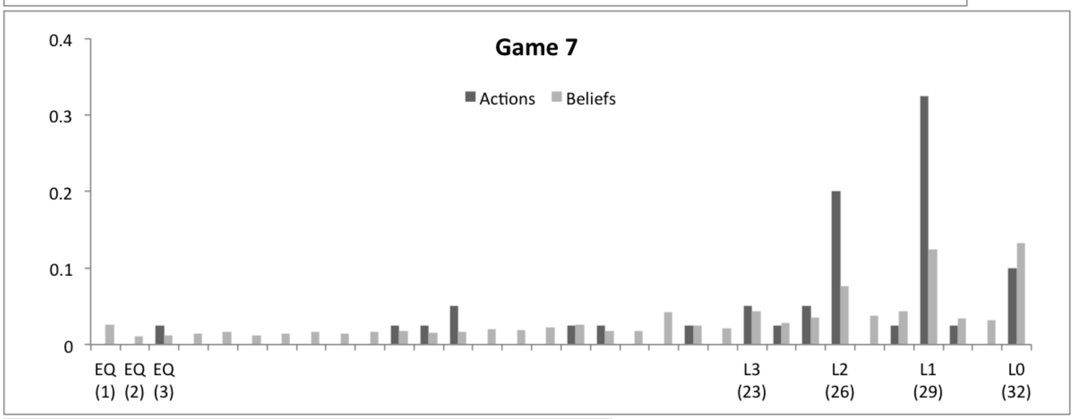


Figure 5.—Relative proportions of decisions in each treatment and game. (Bars in a given chart of a given treatment sum to 1.)





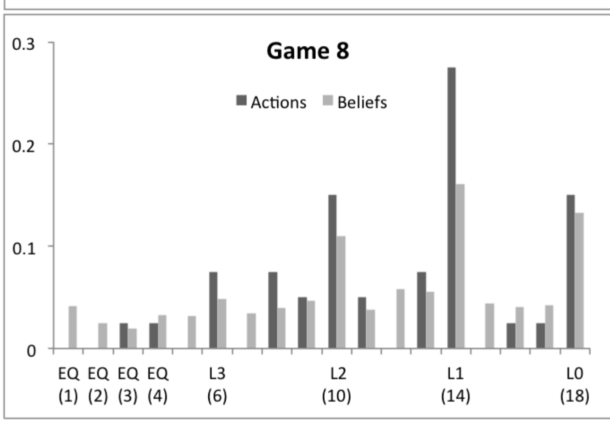


Figure 6.—Relative proportions of decisions in each treatment and game. (Bars in a given chart of a given treatment sum to 1.)

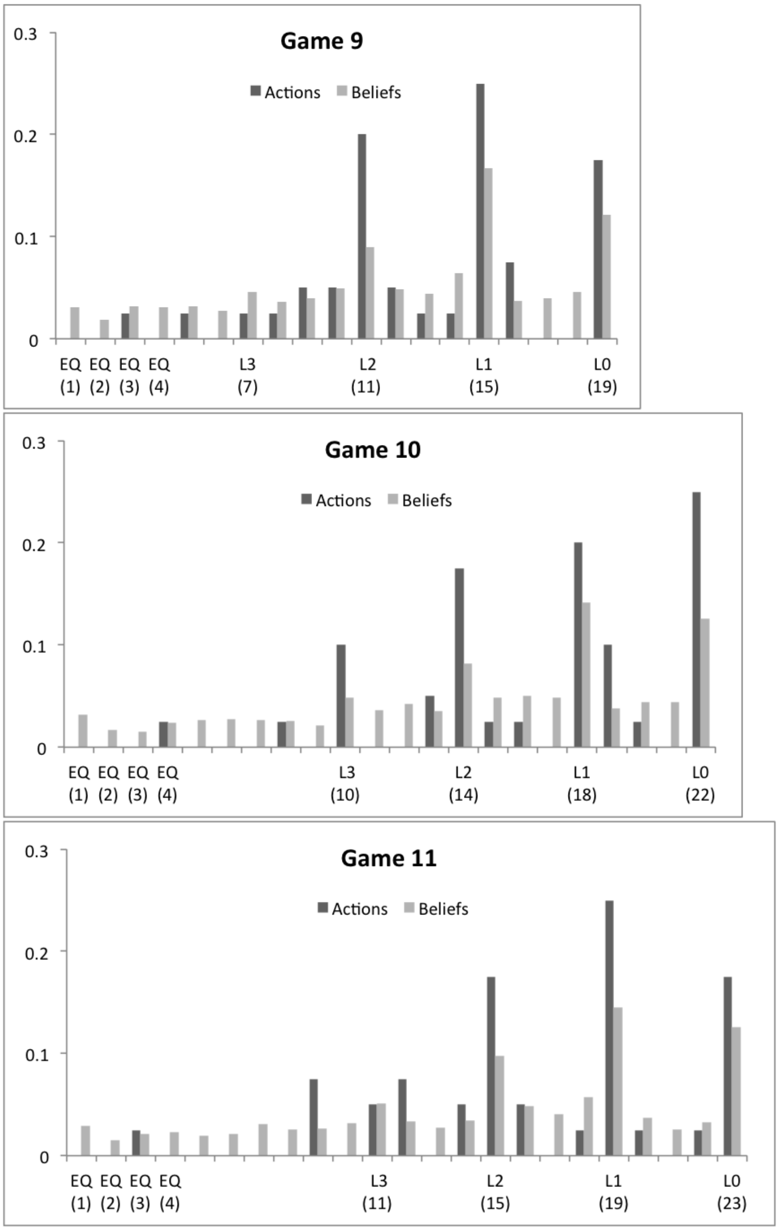


FIGURE 7.—Relative proportions of decisions in each treatment and game. (Bars in a given chart of a given treatment sum to 1.)

Appendix E: Complete Instructions (not intended for publication)

[Paper-Based Instructions for *Actions* Treatment]

Welcome! You are about to participate in an experiment regarding decision-making. The experiment is expected to last approximately 1 hour. If you follow the instructions carefully, your choices may earn you a CONSIDERABLE AMOUNT OF MONEY. Your earnings will be paid to you privately in cash at the end of the experiment. You will receive a $5 show-up payment which is yours to keep. In addition, you can earn more money during the experiment. If you have any questions from now until you are dismissed, please raise your hand and an experimenter will come to you.

Please know that we sincerely appreciate your participation in today's study. Your participation is what makes it possible for us to make progress in our research. As a result, we ask for your cooperation in helping us keep the experimental environment free of any distractions, both to yourself and to others. Specifically, please DO NOT talk, look at others' screens, or exclaim out loud at any point during the study. Please do not use any electronics that you have brought with you (phones, tablets, laptops). Please only use what we have provided to you (pens, paper, calculators, computers).

For the entire duration of today's experiment, you will be matched randomly and anonymously with another participant, henceforth known as "The Other Participant". No participant will ever be told with whom they are matched.

You and The Other Participant will play 11 different GAMES. In each Game, you and The Other Participant will have 2 minutes to separately and independently select NUMBERS.

Together, the Numbers selected by you and The Other Participant determine the POINTS that you earn and the Points that The Other Participant earns. (Your Points and The Other Participant's Points may be different.)

At the end of the experiment, your Points from a Game will be translated into money. You will be paid for all 11 Games. For each Game, you will earn $1 or $5. The more Points you earn in a Game, the higher your chances will be of earning $5 for that Game. Further details about this will be explained later in the instructions.

Within a Game, you can change the Number you select as many times as you wish. However, once you click the 'Confirm' button, the Number selected is permanently recorded for that Game, meaning you can no longer select a different Number for that Game. After clicking the 'Confirm' button, you must wait for all other participants to click their 'Confirm' buttons before you (and everyone else) begins the next Game.

Therefore, since you must wait for everyone else to finish a Game before you can move to the next one, you cannot "race through" the Games. We hope that this will lead you to take your time and carefully think about the Numbers you select.

ARE THERE ANY QUESTIONS THUS FAR?

THE FEATURES OF A GAME: Range and Undercutting Distance

In each Game, there will be a RANGE that is shown identically to you and to The Other Participant. The Range always starts at 1, and goes up to some number, and does not "skip" numbers. For example, if the Range is 11, the Range will look like:

1	2	3	4	5	6	7	8	9	10	11
☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐

(A Range of 11)

In each Game, you will click a box to select a Number from the Range, and The Other Participant will click a box to select a Number from the Range. The selected Numbers are submitted (and finalized) by clicking a 'Confirm' button underneath the Range.

Before each Game and between Games

- you will NOT observe the Numbers selected by The Other Participant, and
- The Other Participant will NOT observe the Numbers you select

In addition to the Range, each Game will possess an UNDERCUTTING DISTANCE that is shown identically to you and to The Other Participant. In a Game, the Points earned by you and by The Other Participant will depend on your selected Number, The Other Participant's selected Number, and the Undercutting Distance.

HOW POINTS ARE EARNED

- You will receive the Number you select, in Points.
- Similarly, The Other Participant will receive the Number they select, in Points.
- You will receive **100 BONUS POINTS** if your Number undercuts The Other Participant's Number by the Undercutting Distance. For example, suppose the Undercutting Distance is 2 and suppose The Other Participant selects the Number 7. Then, if you select 5, you earn 100 Bonus Points. This is because 5 is 2 less than 7, and 2 is the Undercutting Distance.
- Similarly, The Other Participant will receive **100 BONUS POINTS** if their Number undercuts your Number by the Undercutting Distance.
- You and The Other Participant will each receive **35 BONUS POINTS** if you select the **same Numbers**.

To make sure the instructions are clear up to this point, we will provide an example on the next page.

ARE THERE ANY QUESTIONS THUS FAR?

SAMPLE GAME AND EXAMPLES OF HOW POINTS ARE EARNED

(Example of How You and the Other Participant View a Game)

The RANGE is **1 to 15** and the UNDERCUTTING DISTANCE is **4**.

You and The Other Participant are to select Numbers from the Range.

You will receive the Number you select **IN POINTS** and The Other Participant will receive the Number they select **IN POINTS**.

You will receive **100 BONUS POINTS** if your Number is **exactly 4 less** than The Other Participant's Number.

The Other Participant will receive **100 BONUS POINTS** if their Number is **exactly 4 less** than your Number.

If You and The Other Participant select the **same Numbers**, you will each earn **35 BONUS POINTS**.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐

All numbers in the examples below are chosen for illustrative purposes and are NOT meant to suggest how you should select your Numbers in the actual experiment.

Example 1: Suppose you and The Other Participant both select 12 as your Numbers. You will each earn 12 Points. Furthermore, since your Numbers are equal, you will each earn 35 Bonus Points. You will each receive 12 + 35 = 47 Points.

Example 2: Suppose you select the Number 3 and The Other Participant selects the Number 7. You will earn 3 Points and The Other Participant will earn 7 Points. Furthermore, since 4 is the Undercutting Distance and the Number you selected is 4 less than The Other Participant's Number, you will earn 100 Bonus Points. In total, you will receive 3 + 100 = 103 Points and The Other Participant will receive 7 Points.

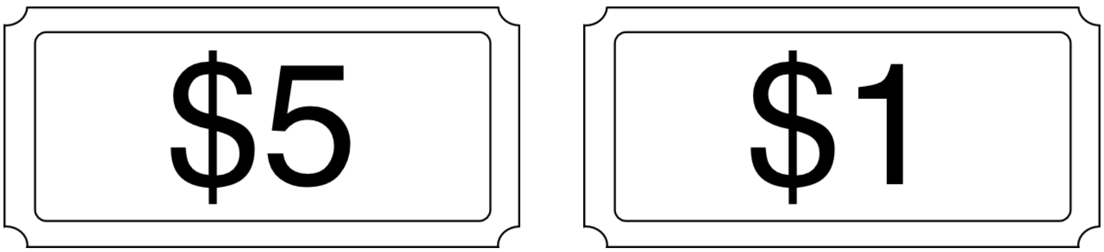
Example 3: Suppose you select the Number 14 and The Other Participant selects the Number 12. You will earn 14 Points and The Other Participant will earn 12 Points. There are no Bonus Points earned by anyone because the Numbers you both selected are not equal, nor is one Number 4 less than the other. Therefore, in total, you will earn 14 Points and The Other Participant will earn 12 Points.

ARE THERE ANY QUESTIONS THUS FAR?

CONVERTING POINTS INTO MONEY: Tickets in Ticket Boxes

At the end of the experiment, your Points from each Game will be converted into money and paid to you. An identical procedure will be used to convert The Other Participant's Points into money which will be paid to them. Since the procedures are identical, we will only explain the procedure in terms of you and your Points.

For each Game, you can earn $5 or $1. The more Points you earn, the higher will be your probability of earning $5. In particular, imagine a Ticket Box that contains **150 Tickets in total**. Some of the Tickets say $5 and some of the Tickets say $1.



The number of Tickets that say $5 is equal to the number of Points you earn in a Game. For example, if you earn 50 Points in a Game, the Ticket Box will contain 50 Tickets that say $5. The remaining 100 Tickets will say $1.

At random, the computer will select a Ticket from the Ticket Box. If the selected Ticket says $5, you earn $5. If the selected Ticket says $1, you earn $1. For example, if you earn 120 Points, then 120 of the Tickets say $5 and the remaining 30 say $1.

Using the Points you earn in the 11 Games, the computer will construct 11 Ticket Boxes, one per Game. Separately and independently, the computer will select 11 Tickets at random, one from each Ticket Box.

You will be paid the total value of the 11 Tickets that are drawn. For example, if the computer selects 7 Tickets that say $5 and 4 Tickets that say $1, you would earn $44 in TOTAL EARNINGS:

$$\overbrace{\$5}^{\text{Show-up payment}} + \overbrace{\$35}^{\textbf{7}\text{ Tickets that say }\textbf{\$5}} + \overbrace{\$4}^{\textbf{4}\text{ Tickets that say }\textbf{\$1}} = \overbrace{\$44}^{\text{Total Earnings}}$$

ARE THERE ANY QUESTIONS THUS FAR?

SUMMARY

You and The Other Participant with whom you are matched will play 11 Games, each of which has a Range of numbers and an Undercutting Distance.

You and The Other Participant will separately select Numbers from the Range. These Numbers, along with the Undercutting Distance, will determine the number of Points you earn and the Number of Points that The Other Participant earns:

- You will earn the Number you select in Points. Similarly, The Other Participant will earn the Number they select in Points.
- If you and The Other Participant select the same Numbers, each of you receives 35 Bonus Points.
- If your Number undercuts The Other Participant's Number by the Undercutting Distance, you will earn 100 Bonus Points.
- Similarly, if The Other Participant's selected Number undercuts your Number by the Undercutting Distance, they will earn 100 Bonus Points.

At the end of the experiment, your Points from a Game will be translated into $1 or $5. The more Points you earn in a Game, the higher your chances will be of earning $5 for that Game.

Specifically, we create 11 Ticket Boxes, one per Game. Each Ticket Box contains 150 Tickets in total. In the Ticket Box corresponding to a particular Game, the number of Tickets that say $5 is equal to the number of Points you earned in that Game.

For example, if you earn 50 Points in a Game, the Ticket Box will contain 50 Tickets that say $5. The remaining 100 Tickets will say $1.

The computer will select a Ticket at random from each Ticket Box and you will be paid the total value of the 11 Tickets that are selected.

ARE THERE ANY QUESTIONS?

On your desk, you should have a calculator, a pen and scratch paper. You are free to use these materials from now until the end of the experiment if you find them helpful in any way. Please raise your hand if you are missing any of these materials.

Before we begin the experiment, you will be given an UNDERSTANDINGS TEST (on your computers) to help you familiarize yourself with the instructions we've just read altogether. When your monitor starts showing you the Understandings Test, please work through it quietly, on your own, and raise your hand if you have any questions.

[On-Screen Understandings Test for ***Actions*** Treatment]

SCREEN 1

Welcome to the Understanding Test! All numbers in the Understandings Test are picked randomly and are for illustrative purposes only. They are not meant to suggest in any way how you should make your Decisions in the actual experiment.

SCREEN 2

Game # 1 of Understandings Test

The "Range" in this Game is **1 to 12** and the "Undercutting Distance" is **3**.

You and The Other Participant are to select Numbers from the Range.

You will receive the Number you select **IN POINTS** and The Other Participant will receive the Number they select **IN POINTS**.

You will receive **100 BONUS POINTS** if your Number is **exactly 3 less** than The Other Participant's Number.

The Other Participant will receive **100 BONUS POINTS** if their Number is **exactly 3 less** than your Number.

If You and The Other Participant select the **same Numbers**, you will each earn **35 BONUS POINTS**.

1	2	3	4	5	6	7	8	9	10	11	12
☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐

–The questions below refer to Game #1 of the Understandings Test (shown above)–

1. If The Other Participant selects the Number 10, what Number would you need to select in Game #1 for you to earn the most POINTS?
   Answer: [ ] *'OK'*

   Appears after the participant clicks 'OK':

   [7] is the correct answer because if your Number is 3 less than The Other Participant's Number, you earn 100 BONUS POINTS. Thus, your total POINTS earned would be 100 + 7 = 107 POINTS.

2. If The Other Participant selects the Number 5, what is the largest number of POINTS that you can earn in Game #1? Answer: [ ] *'OK'*

   Appears after the participant clicks 'OK':

   [102] is the correct answer because if your Number is 3 less than The Other Participant's Number, you earn 100 BONUS POINTS. Thus, if you select 2, your total POINTS earned would be 100 + 2 = 102 POINTS.

SCREEN 3

Game # 2 of Understandings Test

The “Range” in this Game is **1 to 14** and the “Undercutting Distance” is **4**.

You and The Other Participant are to select Numbers from the Range.

You will receive the Number you select **IN POINTS** and The Other Participant will receive the Number they select **IN POINTS**.

You will receive **100 BONUS POINTS** if your Number is **exactly 4 less** than The Other Participant's Number.

The Other Participant will receive **100 BONUS POINTS** if their Number is **exactly 4 less** than your Number.

If You and The Other Participant select the **same Numbers**, you will each earn **35 BONUS POINTS**.

1	2	3	4	5	6	7	8	9	10	11	12	13	14
☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐

–The questions below refer to Game #2 of the Understandings Test (shown above)–

1. If you select the Number 12, what Number would The Other Participant need to select in Game #2 to earn him/her the most POINTS?
   Answer: ☐ *'OK'*

Appears after the participant clicks 'OK':

8 is the correct answer because if The Other Participant's Number is 4 less than yours, they earn 100 BONUS POINTS. Thus, The Other Participant's total POINTS earned would be 100 + 8 = 108 POINTS.

2. If you select the Number 4 and The Other Participant selects 6, how many POINTS would you earn in Game #2? Answer: ☐ *'OK'*

Appears after the participant clicks 'OK':

4 is the correct answer because selecting a Number that is 2 less than The Other Participant's earns you NO BONUS POINTS in this Game. Thus, your total POINTS in earned in Game #2 would be 2 POINTS.

SCREEN 4

Game # 3 of Understandings Test

The “Range” in this Game is **1 to 11** and the “Undercutting Distance” is **5**.

You and The Other Participant are to select Numbers from the Range.

You will receive the Number you select **IN POINTS** and The Other Participant will receive the Number they select **IN POINTS**.

You will receive **100 BONUS POINTS** if your Number is **exactly 5 less** than The Other Participant’s Number.

The Other Participant will receive **100 BONUS POINTS** if their Number is **exactly 5 less** than your Number.

If You and The Other Participant select the **same Numbers**, you will each earn **35 BONUS POINTS**.

1	2	3	4	5	6	7	8	9	10	11
☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐

–The questions below refer to Game #3 of the Understandings Test (shown above)–

1. If You and The Other Participant both select the Number 6 in Game #3, how many POINTS would The Other Participant earn?
   Answer: ☐ *‘OK’*

   Appears after the participant clicks ‘OK’:

   41 is the correct answer because you and The Other Participant would each earn 35 BONUS POINTS for selecting the same Numbers. Thus, The Other Participant would earn a total of 6 + 35 = 41 POINTS.

2. If you select the Number 8 and The Other Participant selects the Number 3, how many POINTS would you earn? Answer: ☐ *‘OK’*

   Appears after the participant clicks ‘OK’:

   8 is the correct answer. If you select a Number that is 5 more than The Other Participant’s Number, while they would earn 100 BONUS POINTS, you would earn NO BONUS POINTS. In total, you would earn 8 POINTS.

SCREEN 5

1. We will construct Ticket Boxes (one for each of the 11 Games). How many Tickets will be inside each Ticket Box? Answer: [ ] *'OK'*

   Appears after the participant clicks 'OK':

   150 is the correct answer.

2. Inside a Ticket Box, there will be tickets that either say $1 or $5. If you earned 120 POINTS in a particular Game, how many Tickets in that Game's corresponding Ticket Box will say **$5**? Answer: [ ] *'OK'*

   Appears after the participant clicks 'OK':

   120 is the correct answer. The Number of $5 Tickets is equal to the number of POINTS earned in the Game.

3. Inside a Ticket Box, there will be tickets that either say $1 or $5. If you earned 50 POINTS in a particular Game, how many Tickets in that Game's corresponding Ticket Box will say **$1**? Answer: [ ] *'OK'*

   Appears after the participant clicks 'OK':

   100 is the correct answer. The Number of $5 Tickets is equal to 50, since you earned 50 POINTS. So, the remaining 100 Tickets will say $1.

[Paper-Based Instructions for *Beliefs* Treatment - INTRODUCTION]

Welcome! You are about to participate in an experiment regarding decision-making. The experiment is expected to last approximately 2 hours. If you follow the instructions carefully, your choices may earn you a CONSIDERABLE AMOUNT OF MONEY. Your earnings will be paid to you privately in cash at the end of the experiment. You will receive a $5 show-up payment which is yours to keep. In addition, you can earn more money during the experiment. If you have any questions from now until you are dismissed, please raise your hand and an experimenter will come to you.

Please know that we sincerely appreciate your participation in today's study. Your participation is what makes it possible for us to make progress in our research. As a result, we ask for your cooperation in helping us keep the experimental environment free of any distractions, both to yourself and to others. Specifically, please DO NOT talk, look at others' screens, or exclaim out loud at any point during the study. Please do not use any electronics that you have brought with you (phones, tablets, laptops). Please only use what we have provided to you (pens, paper, calculators, computers).

Today's experiment consists of THREE PARTS. In each Part, you have the opportunity to earn money. At the end of all three Parts, you will be paid the sum of your earnings from Parts One, Two and Three as well as your $5 show-up payment.

Last week, in this lab, we ran an experiment using 20 ORIGINAL SUBJECTS that we recruited using the same procedures that we used to recruit you for today's study. When these 20 Original Subjects entered the lab, they were immediately split, randomly and anonymously, into distinct groups of size 2. They were made aware of this group formation, but they were NOT told *who* was actually in their group.

Each 2-person group followed an identical procedure. We will describe this procedure to you by using a single group with members that we will call "Jack" and "Jill".

Jack and Jill played 11 different GAMES in last week's experiment. In each Game, there was a RANGE of whole numbers that was shown identically to Jack and to Jill. The Range always started at 1 and went up to some number. The Range did not "skip" numbers. For example, if the Range was 11, it looked like:

1	2	3	4	5	6	7	8	9	10	11
☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐

Upon being presented with a Game, Jack had 2 minutes to select a Number. Similarly, Jill had 2 minutes to select a Number. **Before each Game and between Games, neither Jack nor Jill observed the Numbers that the other person selected**.

Jack and Jill both knew, however, that Jack would receive the Number he selected, **IN POINTS**. So, they knew that if Jack chose the Number 7, he would receive 7 Points.

Analogously, Jack and Jill both knew that Jill would receive the Number she selected, **IN POINTS**. So, they knew that if Jill chose the Number 9, she would receive 9 Points.

In addition, Jack and Jill both knew that Jack would receive **100 BONUS POINTS** if his Number undercut Jill's by a particular UNDERCUTTING DISTANCE that Jack and Jill were shown before they selected their Numbers. For example, suppose the Undercutting Distance in a Game was 2, and suppose Jill selected the Number 7. If Jack were to select 5, he would earn 100 Bonus Points since his Number would be exactly 2 less than Jill's.

Analogously, Jack and Jill both knew that Jill would receive **100 BONUS POINTS** if her Number undercut Jack's by the same-sized Undercutting Distance.

Jack and Jill were also both aware that if they were to select the **same Numbers** in a Game, they would each earn **35 BONUS POINTS**.

Before each Game and between Games, neither Jack nor Jill observed the Numbers that the other person selected. At the end of the experiment, all 20 Original Subjects were paid for all 11 Games. On average, more Points translated into more money.

ARE THERE ANY QUESTIONS THUS FAR?

To make sure the instructions are clear up to this point, let's provide some examples of how the Numbers that Jack and Jill chose determined the Points that they earned. Note: the numbers in the examples below are not meant to suggest how the 20 Original Subjects actually selected their Numbers in the previous experiment. They are for illustrative purposes only.

Example Game #1: Range of **1 to 14** and Undercutting Distance is **3**.
Suppose Jack and Jill both selected 12 as their Numbers. They would each earn 12 Points for selecting 12. Furthermore, since their Numbers were equal, they would each earn 35 Bonus Points. In total, they would each receive 12 + 35 = 47 Points.

Example Game #2: Range of **1 to 9** and Undercutting Distance is **4**.
Suppose Jill selected 3 and Jack selected 7. Jill would earn 3 Points and Jack would earn 7 Points. Furthermore, since 4 is the Undercutting Distance and the Number Jill selected was 4 less than Jack's Number, Jill would earn 100 Bonus Points. In total, Jill would receive 3 + 100 = 103 Points and Jack would receive 7 Points.

Example Game #3: Range of **1 to 15** and Undercutting Distance is **5**.
Suppose Jack selected 14 and Jill selected 12. Jack would earn 14 Points and Jill would earn 12 Points. There would be no Bonus Points earned by either of them because their selected Numbers are not the same, nor is one of their Numbers 5 less than the other. Therefore, in total, Jack would earn 14 Points and Jill would earn 12 Points.

ARE THERE ANY QUESTIONS?

On your desk, you should have a calculator, a pen and scratch paper. You are free to use these materials from now until the end of the experiment if you find them helpful in any way. Please raise your hand if you are missing any of these materials.

Before we begin the experiment, you will be given an UNDERSTANDINGS TEST (on your computers) to help you familiarize yourself with the instructions we've just read altogether. When your monitor starts showing you the Understandings Test, please work through it quietly, on your own, and raise your hand if you have any questions.

[On-Screen Understandings Test #1 for *Beliefs* Treatment]

SCREEN 1

Welcome to the Understanding Test! All numbers in the Understandings Test are picked randomly and are for illustrative purposes only. They are not meant to suggest in any way how you should make his Decisions in the actual experiment.

SCREEN 2

Game # 1 of Understandings Test

The "Range" in this Game is **1 to 12** and the "Undercutting Distance" is **3**.

Jack and Jill selected Numbers from the Range.

Jack received the Number he selected **IN POINTS** and Jill received the Number she selected **IN POINTS**.

Jack received **100 BONUS POINTS** if his Number was **exactly 3 less** than Jill's Number.

Jill received **100 BONUS POINTS** if her Number was **exactly 3 less** than Jack's Number.

If Jack and Jill selected the **same Numbers**, each received **35 BONUS POINTS**.

1	2	3	4	5	6	7	8	9	10	11	12
☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐

–The questions below refer to Game #1 of the Understandings Test (shown above)–

1. If Jill selected the Number 10, what Number would Jack need to select in Game #1 to earn the most POINTS?
   Answer: ☐ 'OK'

   Appears after the clicks 'OK':

   7 is the correct answer because if his Number is 3 less than Jill's Number, Jack would earn 100 BONUS POINTS. Thus, his total POINTS earned would be 100 + 7 = 107 POINTS.

2. If Jill selected the Number 5, what is the largest number of POINTS that Jack can earn in Game #1? Answer: ☐ 'OK'

   Appears after the clicks 'OK':

   102 is the correct answer because if his Number is 3 less than Jill's Number, Jack would earn 100 BONUS POINTS. Thus, if Jack selected 2, his total POINTS earned would be 100 + 2 = 102 POINTS.

SCREEN 3

Game # 2 of Understandings Test

The "Range" in this Game is **1 to 14** and the "Undercutting Distance" is **4**.

Jack and Jill selected Numbers from the Range.

Jack received the Number he selected **IN POINTS** and Jill received the Number she selected **IN POINTS**.

Jack received **100 BONUS POINTS** if his Number was **exactly 4 less** than Jill's Number.

Jill received **100 BONUS POINTS** if her Number was **exactly 4 less** than his Number.

If Jack and Jill selected the **same Numbers**, each received **35 BONUS POINTS**.

1	2	3	4	5	6	7	8	9	10	11	12	13	14
☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐

–The questions below refer to Game #2 of the Understandings Test (shown above)–

1. If Jack select the Number 12, what Number would Jill need to select in Game #2 to earn her the most POINTS?
   Answer: [ ] *'OK'*

Appears after the clicks 'OK':

8 is the correct answer because if Jill's Number is 4 less than Jack's, she would earn 100 BONUS POINTS. Thus, Jill's total POINTS earned would be 100 + 8 = 108 POINTS.

2. If Jack selected the Number 4 and Jill selected 6, how many POINTS would Jack earn in Game #2? Answer: [ ] *'OK'*

Appears after the clicks 'OK':

4 is the correct answer because selecting a Number that is 2 less than Jill's earns Jack NO BONUS POINTS in this Game. Thus, his total POINTS in earned in Game #2 would be 2 POINTS.

SCREEN 4

Game # 3 of Understandings Test

The "Range" in this Game is **1 to 11** and the "Undercutting Distance" is **5**.

Jack and Jill selected Numbers from the Range.

Jack received the Number he selected **IN POINTS** and Jill received the Number she selected **IN POINTS**.

Jack received **100 BONUS POINTS** if his Number was **exactly 5 less** than Jill's Number.

Jill received **100 BONUS POINTS** if her Number was **exactly 5 less** than his Number.

If Jack and Jill selected the **same Numbers**, each received **35 BONUS POINTS**.

1	2	3	4	5	6	7	8	9	10	11
☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐

–The questions below refer to Game #3 of the Understandings Test (shown above)–

1. If Jack and Jill both selected the Number 6 in Game #3, how many POINTS would Jill earn?
   Answer: [ ] *'OK'*

Appears after the clicks 'OK':

41 is the correct answer because Jack and Jill would each earn 35 BONUS POINTS for selecting the same Numbers. Thus, Jill would earn a total of 6 + 35 = 41 POINTS.

2. If Jack selected the Number 8 and Jill selected the Number 3, how many POINTS would Jack earn? Answer: [ ] *'OK'*

Appears after the clicks 'OK':

8 is the correct answer. If Jack selected a Number that is 5 more than Jill's Number, while Jill would earn 100 BONUS POINTS, Jack would earn NO BONUS POINTS. In total, Jack would earn 8 POINTS.

[**Paper-Based Instructions for *Beliefs* Treatment - PART ONE**]

Welcome Back! You have just completed an Understandings Test about the previous experiment. We are now ready to explain what you will do in Part One of today's study.

In Part One of today's study, you will express how you think the 20 Original Subjects selected their Numbers in the Games from last week's experiment.

One at a time, we will show you the Games, just as they were presented to the 20 Original Subjects. Thus, you will see a Game's Range and Undercutting Distance.

When shown a Game, your TASK will be to construct a BOX ARRANGEMENT for that Game. This is best understood by example. Hence, we will now distribute a plastic handout showing an Example Box Arrangement **for a Game with Range 1 to 15**.

When you receive the handout, please place it on top of a blank sheet of scratch paper.

Notice the following features of the Example Box Arrangement:

- There is a row of GREEEN TRIANGLES: you will click them to ADD Boxes
- There is a row of RED TRIANGLES: clicking these REMOVES Boxes (since there are NO blue Boxes on top of the "8" or "13" Numbers from the Game's Range, no blue Boxes can be removed, hence there are no Red Triangles there)
- There is an automatic counter underneath each Number in the Game's Range (you can see that it counts 2 Boxes on top of the Number 1, for example)
- At the very bottom of the Box Arrangement, there is a counter for the total number of blue Boxes that have been placed in the Box Arrangement

Your Box Arrangement will express how you think the 20 Original Subjects chose their Numbers. Hence it will consist of 20 Boxes. For example,

Example 1: To express a belief that 2 Original Subjects chose the Number 1, you would place 2 Boxes in the "1" Column, as shown in the Example Box Arrangement.

Example 2: To express a belief that 3 Original Subjects chose the Number 12, you would place 3 Boxes in the "12" Column, as shown in the Example Box Arrangement.

Example 3: To express a belief that there were NO Original Subjects who chose the Number 12, you would leave the "12" Column empty, as shown.

Important Note: the Example Box Arrangement on your handout was created purely at random and is for illustrative purposes only. It is NOT meant to suggest how the 20 Original Subjects actually chose their Numbers in last week's experiment.

At no point will you be shown any decisions made by the 20 Original Subjects.

ARE THERE ANY QUESTIONS THUS FAR?

You will have 2 minutes to complete each of the 11 Tasks. During a Task, you can edit your Box Arrangement from it as much as you wish. However, once you click 'Confirm', the Box Arrangement you've constructed in the Task is permanently recorded.

Note: after clicking 'Confirm', you must wait for all other participants to click their 'Confirm' buttons before you (and everyone else) begins the next Task. Therefore, it's not possible to "race through" Part One of the experiment. We hope this will lead you to think carefully and take your time when you build your Box Arrangements.

AFTER YOU COMPLETE your 11 Box Arrangements, we will compare them to the actual decisions that were made by the 20 Original Subjects.

Specifically, for each Game, there is a DOT ARRANGEMENT that describes how the 20 Original Subjects actually chose their Numbers in last week's experiment. We will now pass out a second handout that shows an Example Dot Arrangement.

Suppose, for a Game, you built the Example Box Arrangement from the plastic handout, but the 20 Original Subjects actually behaved according to the Dot Arrangement.

Take a moment now, to *physically place* your Example Box Arrangement ON TOP of the Example Dot Arrangement, aligning the thick frames of each arrangement.

Some Boxes are OVERLAPPING with Dots, such as the bottom Box in Column "5". Some Boxes are not Overlapping with Dots, such as the middle Box in Column "12".

To pay you for a Box Arrangement, your computer will place it "on top" of the corresponding true Dot Arrangement of actual behavior by the 20 Original Subjects.

Your computer will then select 1 Box AT RANDOM from your Box Arrangement.

If the selected Box is Overlapping with a Dot, you will earn **$5** from that Box Arrangement. If the selected Box is NOT Overlapping with a Dot, you will earn **$1**.

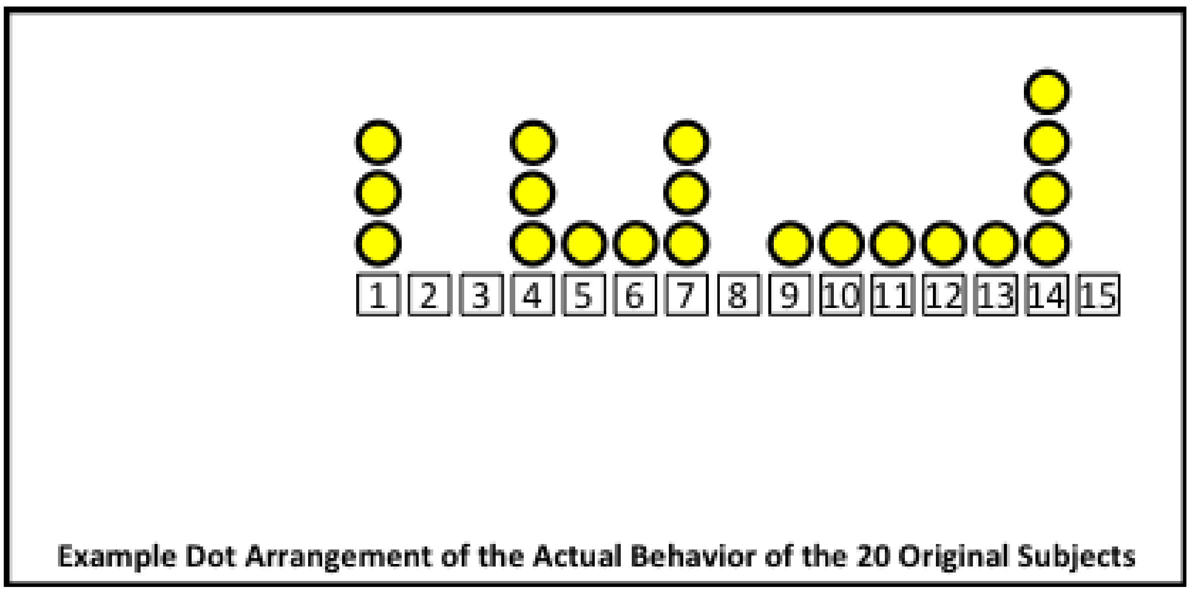
This procedure will be repeated for all 11 Box Arrangements you build. For example, if the computer selects 7 Overlapping Boxes, your Part One earnings would be:

$$\overbrace{\$35}^{\textbf{7}\text{ Overlapping Boxes}} + \overbrace{\$4}^{\textbf{4}\text{ Non-Overlapping Boxes}} = \overbrace{\$39}^{\text{Total Part One Earnings}}$$

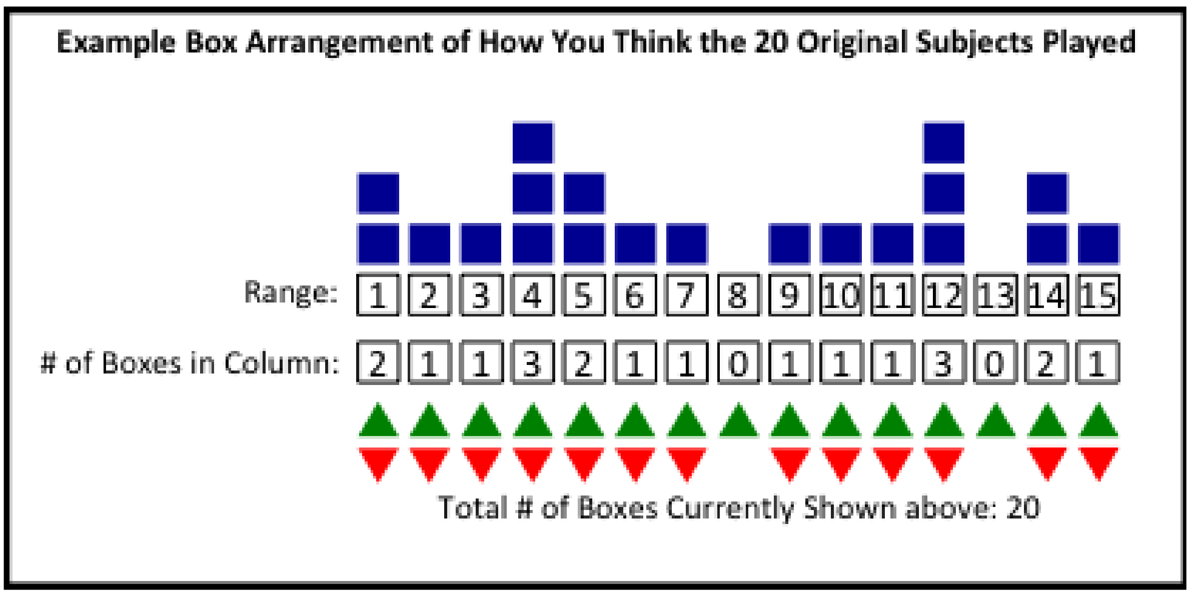
ARE THERE ANY QUESTIONS?

Before we begin Part One of the experiment, you will be given an UNDERSTANDINGS TEST (on your computers) to help you familiarize yourself with these Part One instructions that what we've just read altogether. When your monitor starts showing you the Understandings Test, please work through it quietly, on your own, and raise your hand if you have any questions.

[The framed image below was printed on standard paper and given to subjects]



[The framed image below was printed on plastic transparencies and given to subjects]



[On-Screen Understandings Test #2 for *Beliefs* Treatment]

SCREEN 1

Welcome to the Understandings Test! If you need to look at your instructions to answer some of the questions, please feel free to do so.

SCREEN 2

Question # 1:

How many Original Subjects were there in our experiment last week?

[ ] *'OK'*

(Answer that appears after they click OK: That's right!)

SCREEN 3

Question # 2:

How many Games did the 20 Original Subjects play?

[ ] *'OK'*

(Answer that appears after they click OK: That's right!)

SCREEN 4

Question #3:

How many Tasks will you perform in Part One of today's experiment?

[ ] *'OK'*

(Answer that appears after they click OK: That's right! Each of the 11 Tasks will correspond to one of the 11 Games that the Original Subjects played in last week's experiment.)

SCREEN 5

Question #4:

In a Task, you will state how you think the 20 Original Subjects selected their Numbers in the Game associated with that Task.

In a Task, suppose you wish to express your belief that 4 people chose the Number 6 in the Game associated with that Task.

How would you express this belief in your Box Arrangement for that Task?

a) By creating a stack of 4 Boxes in the “4” Column of the Game’s Range.

b) By creating a stack of 4 Boxes in the “6” Column of the Game’s Range.

c) By creating a stack of 6 Boxes in the “4” Column of the Game’s Range.

d) By creating a stack of 6 Boxes in the “6” Column of the Game’s Range.

(Answer that appears after they select the correct answer: That’s right!)

SCREEN 6

Question #5:

In a Task, you will state how you think the 20 Original Subjects selected their Numbers in the Game associated with that Task.

In a Task, suppose you wish to express your belief that 7 people chose the Number 3 in the Game associated with that Task.

How would you express this belief in your Box Arrangement for that Task?

a) By creating a stack of 3 Boxes in the “3” Column of the Game’s Range.

b) By creating a stack of 3 Boxes in the “7” Column of the Game’s Range.

c) By creating a stack of 7 Boxes in the “3” Column of the Game’s Range.

d) By creating a stack of 7 Boxes in the “7” Column of the Game’s Range.

(Answer that appears after they select the correct answer: That’s right!)

SCREEN 7

Question #6:

Look at the Box Arrangement and Dot Arrangement pictured in your printed-out instructions.

How many Overlapping Boxes are there in the "12" Column?

[ ] *'OK'*

(Answer that appears after they click OK: 1 Overlapping Box is indeed the correct answer! In the "12" Column, there are 3 Boxes in the Box Arrangement and there is 1 Dot in the Dot Arrangement. So, if these columns were placed one on top of the other, there would be 1 Overlapping Box.)

SCREEN 8

Question #7:

Look at the Box Arrangement and Dot Arrangement pictured in your printed-out instructions.

How many Overlapping Boxes are there in the "14" Column?

[ ] *'OK'*

(Answer that appears after they click OK: 2 Overlapping Boxes is indeed the correct answer! In the "14" Column, there are 2 Boxes in the Box Arrangement and there are 4 Dots in the Dot Arrangement. So, if these columns were placed one on top of the other, there would be 2 Overlapping Boxes.)

SCREEN 9

Question #8:

We will pay you for all 11 Box Arrangements.

When we pay you for a particular Box Arrangement, we will award you $5 or $1.

We do this by randomly selecting a Box from your Box Arrangement.

If the selected box is an Overlapping Box, how many dollars would you earn?

[ ] $`OK'$

(Answer that appears after they click OK: That's right!)

[**Paper-Based Instructions for *Beliefs* Treatment - PART TWO**]

Welcome Back! You have just completed the 11 Tasks from Part One of today's.

Specifically, you constructed Box Arrangements to express how you thought the 20 Original Subjects played the Games in last week's experiment.

We will soon begin Part Two of today's experiment, where you will answer 11 PERFORMANCE QUESTIONS, one corresponding to each Task from Part One.

A Performance Question will Read: *"For this Task, how many OTHER participants presently in the lab do you think built Box Arrangements having* ***STRICTLY MORE*** *OVERLAPPING BOXES compared to your Box Arrangement?"* Answer:

Recall the definition of an OVERLAPPING BOX: When the computer places a Box Arrangement for a Game on top of the Dot Arrangement of actual behavior in that Game, an Overlapping Box is one that overlaps with a Dot.

When we ask you a Performance Question for a given Box Arrangement,

1. we will show you the Range and Undercutting Distance for the Game associated with that Box Arrangement
2. we will show you the Box Arrangement that you built in Part One, but NOT any Box Arrangements made by any other participants
3. we will inform you of the total number of other participants presently in the lab

For each Performance Question, you will earn $5 for a correct answer and $1 for an incorrect answer. Only at the end of the experiment will you be informed as to whether you correctly or incorrectly answer Performance Questions.

Note: when you 'Confirm' your answer to a Performance Question, you must wait for all other participants to click their 'Confirm' buttons before you (and everyone else) begins the next Performance Question.

Therefore, it's not possible to "race through" Part Two of the experiment. We hope this will lead you to think carefully and take your time when you answer your Performance Questions.

ARE THERE ANY QUESTIONS?

Before we begin Part Two of the experiment, you will be given an UNDERSTANDINGS TEST (on your computers) to help you familiarize yourself with these Part Two instructions that what we've just read altogether. When your monitor starts showing you the Understandings Test, please work through it quietly, on your own, and raise your hand if you have any questions.

[**On-Screen Understandings Test #3 for *Beliefs* Treatment**]

SCREEN 1

Welcome to the Understandings Test! If you need to look at your instructions to answer some of the questions, please feel free to do so.

SCREEN 2

Question # 1:

How many Tasks did you just complete?

[   ] 'OK'

(Answer that appears after they click OK: That's right!)

SCREEN 3

Question #2:

How many Performance Questions will you perform in Part Two of today's experiment?

[   ] 'OK'

(Answer that appears after they click OK: That's right! Each of the 11 Performance Questions will correspond to one of the 11 Tasks that you completed in Part One.)

SCREEN 4

Question #3:

Imagine there were 5 total participants in today's experiment: Bob, Lynn, Alice, Lea and Dave.

Suppose that, in the Task corresponding to a given Performance Question, the participants have the following amounts of Overlapping Boxes in each of their Box Arrangements:

Bob has 11
Lynn has 8
Alice has 6
Lea has 6
Dave has 3

(These numbers of Overlapping Boxes were picked at random and are for illustrative purposes only.)

How many students have STRICTLY MORE Overlapping Boxes than Lea?

[ ] *'OK'*

(Answer that appears after they select the correct answer: That's right!)

SCREEN 5

Question #4:

Suppose that you answer a Performance Question **correctly**.

In other words, you **correctly** state the number of subjects in today's study that have STRICTLY MORE Overlapping Boxes in their Box Arrangements.

How many dollars will you earn for this Performance Question?

[ ] *'OK'*

(Answer that appears after they select the correct answer: That's right!)

SCREEN 6

Question #5:

Suppose that you answer a Performance Question **incorrectly**.

In other words, you **incorrectly** state the number of subjects in today's study that have STRICTLY MORE Overlapping Boxes in their Box Arrangements.

How many dollars will you earn for this Performance Question?

[ ] 'OK'

(Answer that appears after they select the correct answer: That's right!)

[On-Screen Instructions for *Beliefs* Treatment - PART THREE]

You will be shown 100 closed boxes on your screen. Inside one of these boxes, there is a “bomb”. The bomb has an equal chance of being inside any box.

You will tell the computer how many boxes you would like to open. The computer will open your desired number of boxes and it will choose, at random, which boxes it opens.

Your earnings from the second stage will be determined as follows:

If one of the boxes that the computer opens contains the bomb, you will earn $0 in the second stage.

If the computer does NOT open the box containing the bomb, your second stage earnings will be $0.10 per opened box.

How many boxes do you want the computer to open? [ ]